

THE OSCILLATING WING IN A SUBSONIC FLOW.

R 53, Int 7.

Computation Department Mathematical Centre.

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INTRODUCTION

This report R 53, Int 7 is the seventh of a number of interim reports giving information about computations carried out by the Computation Department of the Mathematical Centre on behalf of the National Aeronautical Research Institute in Amsterdam under contract R 53. The final report R 53 that will be made up eventually will not contain much else than the final results of the computations and, moreover, will be not available for general distribution. As however in the course of the computations a lot of information has to be compiled for internal use, and part of this information may be of some value to others, this compilation will be done in the form of interim reports, that will be made available for limited circulation.

Summary.

Section 2 of this report deals with the computation of the function

$$L_1(\Omega, \eta_1) = \sin \eta_1 \int_0^\infty e^{-i\Omega \cosh \xi} \frac{d\xi}{\cosh \xi + \cos \eta_1}. \quad (1,1)$$

The other sections are devoted to the summation of the following series

$$\sum_{n=1}^{\infty} G_n(\beta, \Omega, \eta_1), \quad (1,2)$$

$$\sum_{n=1}^{\infty} \left\{ \operatorname{se}_n(\eta_1) \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} + \sum_{m=1}^{\infty} B_m^{(n)} \frac{\sin m\eta_1}{m} \right\} S_n(\beta, \Omega, \eta_1), \quad (1,3)$$

$$\sum_{n=1}^{\infty} \left\{ \operatorname{se}_m(\eta_1) \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} + \sum_{m=1}^{\infty} B_m^{(n)} \frac{\sin m\eta_1}{m} \right\} T_n(\beta, \Omega, \eta_1), \quad (1,4)$$

$$\sum_{n=1}^{\infty} \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} S_n(\beta, \Omega, \eta_1) S_n^*(\beta, \Omega, \eta_1), \quad (1,5)$$

$$\sum_{n=1}^{\infty} \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} S_n(\beta, \Omega, \eta_1) T_n^*(\beta, \Omega, \eta_1), \quad (1,6)$$

$$\sum_{n=1}^{\infty} \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} S_n^*(\beta, \Omega, \eta_1) T_n(\beta, \Omega, \eta_1), \quad (1,7)$$

$$\sum_{n=1}^{\infty} \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} T_n(\beta, \Omega, \eta_1) T_n^*(\beta, \Omega, \eta_1), \quad (1,8)$$

$$\sum_{n=1}^{\infty} \Lambda_n(\beta, \Omega) S_n^*(\beta, \Omega, \eta_1) \quad (1,9)$$

$$\sum_{n=1}^{\infty} \Lambda_n(\beta, \Omega) T_n^*(\beta, \Omega, \eta_1). \quad (1,10)$$

The functionsymbols are explained in former reports, and the asterisk denotes the complex conjugated. Section 3 gives the method used, section 4 some expansions, and, finally, results are given in sections 5 through 12.

2. Computation of $L_1(\Omega, \eta_1)$.

From the definition (7,15) of the report F 54 of the National Aeronautical Research Institute, viz.

$$L_1(\Omega, \eta_1) = \sin \eta_1 \int_0^{\infty} e^{-i\Omega \cosh \xi} \frac{d\xi}{\cosh \xi + \cos \eta_1}, \quad (2,1)$$

one sees that the integral converges for values of Ω with a non-positive imaginary part. If one chooses the imaginary part of Ω to be negative then it is easily proved by means of

$$\frac{\partial L_1(\Omega, \eta_1)}{\partial \Omega} = -i \sin \eta_1 \int_0^{\infty} e^{-i\Omega \cosh \xi} \frac{\cosh \xi d\xi}{\cosh \xi + \cos \eta_1}$$

that $L_1(\Omega, \eta_1)$ satisfies the differential equation of the first order:

$$\frac{dL_1(\Omega, \eta_1)}{d\Omega} - i \cos \eta_1 L_1(\Omega, \eta_1) = -\frac{\pi}{2} \sin \eta_1 H_0^{(2)}(\Omega). \quad (2,2)$$

Solving this equation one has

$$L_1(\Omega, \eta_1) = e^{i\Omega \cos \eta_1} \left\{ \eta_1 - \frac{\pi}{2} \sin \eta_1 \int_0^{\Omega} e^{-i \cos \eta_1 t} H_0^{(2)}(t) dt \right\}, \quad (2,3)$$

where the following identity is used:

$$L_1(0, \eta_1) = \sin \eta_1 \int_0^{\infty} \frac{d\xi}{\cosh \xi + \cos \eta_1} = 2 \int_0^{\infty} \frac{dt}{\frac{1+\cos \eta_1}{\sin \eta_1} + t^2} = \eta_1. \quad (2,4)$$

Formula (2,3) can be justified also for real values of Ω by means of analytic continuation.

Putting $c = \cos \eta_1$, and using

$$A = \int_0^{\Omega} \cos ct J_0(t) dt - \int_0^{\Omega} \sin ct Y_0(t) dt, \quad (2,5)$$

$$B = \int_0^{\Omega} \sin ct J_0(t) dt + \int_0^{\Omega} \cos ct Y_0(t) dt, \quad (2,6)$$

yields

$$\operatorname{Re} L_1(\Omega, \eta_1) = \sin \eta_1 \left\{ \frac{\eta_1}{\sin \eta_1} \cos c\Omega - \frac{\pi}{2} A \cos c\Omega - \frac{\pi}{2} B \sin c\Omega \right\}, \quad (2,7)$$

$$\operatorname{Im} L_1(\Omega, \eta_1) = \sin \eta_1 \left\{ \frac{\eta_1}{\sin \eta_1} \sin c\Omega - \frac{\pi}{2} A \sin c\Omega + \frac{\pi}{2} B \cos c\Omega \right\}, \quad (2,8)$$

So the problem is now to compute A and B. Following L. Schwarz (Luftfahrtforschung 1944; 341-372) the following functions are defined:

$$\begin{aligned} J_c(\lambda, x) &= \int_0^x J_0(\lambda t) \cos t \, dt, \\ J_s(\lambda, x) &= \int_0^x J_0(\lambda t) \sin t \, dt, \\ Y_c(\lambda, x) &= \int_0^x Y_0(\lambda t) \cos t \, dt, \\ Y_s(\lambda, x) &= \int_0^x Y_0(\lambda t) \sin t \, dt. \end{aligned} \tag{2,9}$$

Hence,

$$\begin{aligned} A &= \left\{ J_c\left(\frac{1}{c}, \frac{\Omega}{c}\right) - Y_s\left(\frac{1}{c}, \frac{\Omega}{c}\right) \right\}, \\ B &= \left\{ J_s\left(\frac{1}{c}, \frac{\Omega}{c}\right) + Y_c\left(\frac{1}{c}, \frac{\Omega}{c}\right) \right\}. \end{aligned}$$

Schwarz tabulated the functions (2,9) for $\lambda = 0.1(0.1)1$. However, it is easily seen that here $\lambda = 1/c > 1$. Therefore the Computation Department tabulated the functions (2,9) also for values of $\lambda > 1$.

The computations are all made by numerical integration. Only, there rises a difficulty because of the singular part of $Y_0(t)$, viz. $\frac{2}{\pi} J_0(t) \log t$. When one integrates numerically the functions $Y_0(t) \sin ct$ and $Y_0(t) \cos ct$, one has to compute also the error caused by that singular term during the process of integration.

This error is found as follows:

Expand $J_0(t) \cos ct$ and $J_0(t) \sin ct$ into a power-series with respect to t . Now one first considers the error caused by numerical integration in $\int_0^\Omega t^n \log t \, dt$, $n = 0, 1, \dots$, when exactly the same numerical process is used as is actually done in the numerical integration of the functions considered.

This error be $E_n(\Omega)$. $E_n(\Omega)$ is easily computed, because one can evaluate $\int_0^\Omega t^n \log t \, dt$ analytically.

The error caused by the singular term of $Y_0(t)$ in the integral $\int_0^\Omega \cos ct \, Y_0(t) dt$ resp. $\int_0^\Omega \sin ct \, Y_0(t) dt$ is given by

$$\frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{n! n! 2^{2n}} \sum_{k=0}^{\infty} \frac{(-1)^k c^{2k}}{(2k)!} E_{2k+2n}(\Omega),$$

resp.

$$\frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{n! n! 2^{2n}} \sum_{k=0}^{\infty} \frac{(-1)^k c^{2k+1}}{(2k+1)!} E_{2k+2n+1}(\Omega).$$

Both series are highly convergent and only three or four terms are needed in order to yield a result correct in 8 decimals.

This process, by the way, is valid for all linear operations in numerical analysis as e.g. interpolation, integration and differentiation.

A check can be made in another way, for

$$\begin{aligned} L_1(\alpha, \eta_1) &= \sin \eta_1 \int_0^\infty e^{-i\alpha \cosh \xi} \frac{d\xi}{\cosh \xi + \cos \eta_1} = \\ &= 2 \sin \eta_1 \int_1^\infty \frac{e^{-i\frac{\alpha}{2}(x+\frac{1}{x})}}{1+2cx+x^2} dx, \end{aligned} \quad (2,10)$$

where $c = \cos \eta_1$,

$$x = e^\xi, \quad \frac{\alpha}{2} = \alpha.$$

Be γ and γ^* the roots of the equation:

$$x^2 + 2cx + 1 = 0.$$

Hence, γ and γ^* are on the unit-circle. Indeed,

$$\begin{aligned} \gamma &= -\cos \eta_1 + \sqrt{\cos^2 \eta_1 - 1} = -\cos \eta_1 + i \sin \eta_1 = -e^{-i\eta_1}, \\ \gamma^* &= -\cos \eta_1 - i \sin \eta_1 = -e^{i\eta_1}. \end{aligned}$$

So one has :

$$\frac{\gamma^n - \gamma^{*n}}{\gamma - \gamma^*} = (-1)^{n+1} \frac{e^{in\eta_1} - e^{-in\eta_1}}{e^{i\eta_1} - e^{-i\eta_1}} = (-1)^{n+1} \frac{\sin n\eta_1}{\sin \eta_1}.$$

Now one proceeds to compute the integral (2,10) as follows

$$\begin{aligned} \int_1^\infty e^{i\alpha x} \frac{e^{\frac{i\alpha}{x}}}{(x-\gamma)(x-\gamma^*)} dx &= \int_1^\infty e^{i\alpha x} \frac{\sum_{h=0}^\infty \frac{(i\alpha)^h}{x^h h!}}{(x-\gamma)(x-\gamma^*)} dx \\ &= \int_1^\infty e^{i\alpha x} \frac{1}{\gamma - \gamma^*} \left\{ \sum_{n=0}^\infty \frac{\gamma^n}{x^{n+1}} - \sum_{n=0}^\infty \frac{\gamma^{*n}}{x^{n+1}} \right\} \sum_{h=0}^\infty \frac{(i\alpha)^h}{x^h h!} dx \\ &= \int_1^\infty e^{i\alpha x} \sum_{j=0}^\infty x^{-j-1} dx, \end{aligned}$$

with

$$A_j = \sum_{n=1}^j (-1)^{n+1} \frac{\sin n\eta_1}{\sin \eta_1} \frac{(i\alpha)^{j-n}}{(j-n)!}. \quad (2,11)$$

Hence,

$$\int_1^{\infty} \frac{e^{i\alpha x}}{x^j} dx = \frac{e^{i\alpha}}{i\alpha} \sum_{h=1}^{j-1} \frac{(j-h-1)!}{(j-1)!} (i\alpha)^h + \frac{(i\alpha)^{j-1}}{(j-1)!} \int_1^{\infty} \frac{e^{i\alpha x}}{x} dx. \quad (2,12)$$

This result is obtained by means of partial integration. Also:

$$L_1(\Omega, \eta_1) = \sin \eta_1 \cdot 2e^{i\alpha} \left\{ \sum_{k=1}^{\infty} \frac{1}{(i\alpha)^k} \left[\sum_{j=k}^{\infty} A_j \frac{(i\alpha)^j}{j!} \right] (k-1)! \right\} + \\ + 2 \sin \eta_1 \sum_{j=1}^{\infty} A_j \frac{(i\alpha)^j}{j!} \int_1^{\infty} \frac{e^{i\alpha x}}{x} dx. \quad (2,13)$$

The terms of the series can be computed in the following way:

$$\frac{e^{i\alpha} n!}{(i\alpha)^{n+1}} \sum_{j=n+1}^{\infty} A_j \frac{(i\alpha)^j}{j!} = \frac{e^{i\alpha} n!}{(i\alpha)^{n+1}} \left[- \sum_{k=1}^{\infty} \frac{\sin k\eta_1}{\sin \eta_1} i^k J_k(\Omega) - \right. \\ \left. - \sum_{\kappa=1}^n \sum_{l=0}^{n-\kappa} \frac{\sin k\eta_1}{\sin \eta_1} (-1)^{n+1} \frac{(i\alpha)^{2l+k}}{l!(l+k)!} \right] \quad (2,14)$$

$$\sum_{j=1}^{\infty} A_j \frac{(i\alpha)^j}{j-1!} = - \sum_{n=1}^{\infty} i^n \frac{\sin n\eta_1}{\sin \eta_1} J_n(\Omega). \quad (2,15)$$

By means (2,13), (2,14) and (2,15) the value of $L_1(\Omega, \eta_1)$ was checked for one value of Ω and found to be correct in 7 decimals. This served as an independent check against systematic errors.

3. The method of summation.

All series mentioned in section 1 are convergent but the convergence is very poor. In order to overcome this difficulty, the following method was used.

1) An expansion is found for the terms of the series such that a few terms of this expansion give enough accuracy for a fixed value of n . So, e.g. the terms of the series (1,3) and (1,4) are expanded with respect to η_1 , and the first two terms are sufficient to get the terms correct in 5 decimal places.

2) This expansion is summed analytically from that value of n till infinity.

3) Numerically the first n terms of the series are computed and also the result of 2). The sum of both gives a result correct in 5.

Now, it may be observed that all required expansions can be split up into partial fractions, and so can be expressed by means of the following auxiliary functions

$$F(\zeta, \alpha) = \sum_{n=0}^{\infty} \frac{e^{i\zeta(n+\frac{\alpha}{2})}}{2n+\alpha}, \quad (3,1)$$

$$G(\zeta, \alpha) = \sum_{n=0}^{\infty} (-1)^n \frac{e^{i\zeta(n+\alpha)}}{n+\alpha}, \quad (3,2)$$

$$\begin{aligned} C_2^{(2k)}(2, \zeta) &= \sum_{n=1}^{\infty} \frac{\cos 2n\zeta}{(2n)^2} = \frac{1}{4} \zeta^2 - \frac{\pi}{4} \zeta + \frac{\pi^2}{24} - \frac{\cos 2\zeta}{4} - \\ &\quad - \frac{\cos 4\zeta}{16} \dots - \frac{\cos 2k\zeta}{(2k)^2} \quad (0 \leq \zeta \leq \pi), \end{aligned} \quad (3,3)$$

$$\begin{aligned} S_2^{(2k)}(2, \zeta) &= \sum_{n=1}^{\infty} \frac{\sin 2n\zeta}{(2n)^2} = -\frac{\zeta}{2} \ln(2 \sin \zeta) + \frac{1}{2} \int_0^{\zeta} x \cotg x dx - \\ &\quad - \frac{\sin 2\zeta}{4} - \dots - \frac{\sin 2k\zeta}{(2k)^2}, \quad (0 \leq \zeta \leq \pi), \end{aligned} \quad (3,4)$$

$$\begin{aligned} C_3^{(2k)}(2, \zeta) &= \sum_{n=1}^{\infty} \frac{\cos 2n\zeta}{(2n)^3} = \frac{\zeta^2}{4} \ln(2 \sin \zeta) - \frac{\zeta}{2} \int_0^{\zeta} x \cotg x dx + \\ &\quad + \frac{1}{4} \int_0^{\zeta} x^2 \cotg x dx + \frac{1}{3} \zeta(3) - \frac{\cos 2\zeta}{2^3} - \dots - \frac{\cos 2k\zeta}{(2k)^3}, \quad (0 \leq \zeta \leq \pi), \end{aligned} \quad (3,5)$$

$$\begin{aligned} S_3^{(2k)}(2, \zeta) &= \sum_{n=1}^{\infty} \frac{\sin 2n\zeta}{(2n)^3} = \frac{\zeta^3}{12} - \frac{\pi \zeta^2}{8} + \frac{\pi^2}{24} \frac{\sin 2\zeta}{2} - \dots - \frac{\sin 2k\zeta}{(2k)^3}, \\ &\quad (0 \leq \zeta \leq \pi), \end{aligned} \quad (3,6)$$

$$\begin{aligned} \overline{C}_2^{(k)}(\zeta) &= \sum_{n=1}^{\infty} (-1)^n \frac{\cos n\zeta}{n^2} = \frac{\zeta^2}{4} - \frac{\pi^2}{12} + \frac{\cos \zeta}{1} \frac{\cos 2\zeta}{2^2} \dots + \\ &\quad + (-1)^{k+1} \frac{\cos k\zeta}{k^2}, \quad (0 \leq \zeta \leq \pi), \end{aligned} \quad (3,7)$$

$$\begin{aligned} \overline{S}_2^{(k)}(\zeta) &= \sum_{n=1}^{\infty} (-1)^n \frac{\sin n\zeta}{n^2} = -\frac{\zeta}{2} \ln(2 \cos \frac{\zeta}{2}) + \int_0^{\zeta} x \cotg x dx - \\ &\quad - 2 \int_0^{\zeta/2} x \cotg x dx + \frac{\sin \zeta}{1^2} - \frac{\sin 2\zeta}{2^2} + \dots + (-1)^{k+1} \frac{\sin k\zeta}{k^2}, \\ &\quad (0 \leq \zeta \leq \pi), \end{aligned} \quad (3,8)$$

$$\begin{aligned} \overline{S}_3^{(k)}(\zeta) &= \sum_{n=1}^{\infty} (-1)^n \frac{\sin n\zeta}{n^3} = \frac{\zeta^3}{12} - \frac{\pi^2 \zeta}{12} + \frac{\sin \zeta}{1} - \frac{\sin 2\zeta}{2^3} + \dots + \\ &\quad + (-1)^{k+1} \frac{\sin k\zeta}{k^3}, \quad (0 \leq \zeta \leq \pi). \end{aligned} \quad (3,9)$$

The function (3,1) and (3,2) can be dealt with as follows:

$$F(\zeta, \alpha) = \sum_{n=0}^{\infty} \frac{e^{i\zeta(n+\frac{\alpha}{2})}}{2n+\alpha}, \quad \alpha \geq -9,$$

$$F_{\zeta}(\zeta, \alpha) = \frac{i}{2} \sum_{n=0}^{\infty} e^{i\zeta(n+\frac{\alpha}{2})} = \frac{ie^{i\zeta(5+\frac{\alpha}{2})}}{2(1-e^{i\zeta})}.$$

Also,

$$F(\zeta, \alpha) = \int_{-\pi}^{\zeta} \frac{ie^{iz(5+\frac{\alpha}{2})}}{2(1-e^{iz})} dz + F(\pi, \alpha). \quad (3,10)$$

In the same way,

$$G(\zeta, \alpha) = \int_0^{\zeta} \frac{ie^{i(\alpha+8)z}}{1+e^{iz}} dz + G(0, \alpha). \quad (3,11)$$

So it is easily proved that for l being a positive integer

$$F(\zeta, \alpha) = -\frac{1}{2} \left[\ln \left(2 \sin \frac{\zeta}{2} \right) + i \left(\frac{\zeta}{2} - \frac{\pi}{2} \right) \right] - \frac{1}{2} \sum_{k=1}^l \frac{1}{k} e^{ki\zeta},$$

when $\alpha = -8 + 2l$; (3,12)

$$F(\zeta, \alpha) = \frac{1}{2} \ln \left(\cotg \frac{\zeta}{4} \right) + \frac{\pi i}{4} - \sum_{k=1}^l \frac{e^{(2k+1)i\zeta/2}}{2k+1},$$

when $\alpha = -8 + 2l + 1$; (3,13)

$$G(\zeta, \alpha) = (-1)^l \left\{ \left[\ln \left(2 \cos \frac{\zeta}{2} \right) + \frac{i\zeta}{2} \right] + \sum_{k=1}^l (-1)^k \frac{e^{kiz}}{k} \right\},$$

when $\alpha = -7 + l$. (3,14)

These auxiliary functions were computed in 3 figures.

4. Some expansions.

First it is necessary to give expansions for

$S_n(\beta, \Omega, \eta_1)$ and $F_n(\beta, \Omega, \eta_1)$ (See for the definitions R 53. Int 1).

Putting

$$A = e^{i\beta^2 \Omega \cos \eta_1}, \quad (4,1)$$

one sees easily:

$$R_n(\beta^2, \Omega, \eta_1) = \int_0^{\eta_1} e^{i\beta^2 \Omega \cos \eta} \cos n\eta d\eta =$$

$$\begin{aligned}
&= \frac{A \sin n\eta_1}{n} - \frac{i\beta^2\Omega}{2n} \int_0^{\eta_1} e^{i\beta^2\Omega \cos \eta} [\cos (n+1)\eta - \cos(n-1)\eta] d\eta = \\
&= \frac{A \sin n\eta_1}{n} - \frac{i\beta^2\Omega}{2n} [R_{n+1}(\beta^2\Omega, \eta_1) - R_{n-1}(\beta^2\Omega, \eta_1)]. \quad (4,2)
\end{aligned}$$

Calculating $R_{n+1}(\beta^2\Omega, \eta_1)$ and $R_{n-1}(\beta^2\Omega, \eta_1)$ by means of (4,2) and substituting into (4,2), yields

$$\begin{aligned}
R_n(\beta^2\Omega, \eta_1) \left\{ 1 + \left(\frac{\beta^2\Omega}{2}\right)^2 \frac{2}{(n+1)n(n-1)} \right\} = A \left\{ \frac{\sin n\eta_1}{n} - \right. \\
\left. - \frac{i\beta^2\Omega}{2} \left[\frac{\sin(n+1)\eta_1}{(n+1)n} - \frac{\sin(n-1)\eta_1}{n(n-1)} \right] \right\} - \left(\frac{\beta^2\Omega}{2}\right)^2 \left[\frac{R_{n+2}(\beta^2\Omega, \eta_1)}{(n+1)n} - \right. \\
\left. - \frac{R_{n-2}(\beta^2\Omega, \eta_1)}{n(n-1)} \right]. \quad (4,3)
\end{aligned}$$

It is clear that after performing this process $\lceil \log n+1 \rceil = p$ times, the expansion will diverge because of the factors in the denominator. Using only m steps ($m < p$) one gets a good approximation for $R_n(\beta^2\Omega, \eta_1)$, when n is supposed to be large and it is even possible to state the order of the remainder term, viz. $O(n^{-2m-1}(\frac{\beta^2\Omega}{2})^{2m+1}\eta_1)$.

Neglecting the terms of orders lower than n^{-2} one finds the following expansions:

$$\begin{aligned}
S_n(\beta^2\Omega, \eta_1) &= \int_0^{\eta_1} e^{i\beta^2\Omega \cos \eta} \operatorname{se}_n(\eta) \sin \eta d\eta = \frac{1}{2} \sum_{r=1}^{\infty} B_r^{(n)} \\
&\quad \{ R_{r-1}(\beta^2\Omega, \eta_1) - R_{r+1}(\beta^2\Omega, \eta_1) \} = \\
&= \frac{A}{2} \left[\left\{ \frac{\sin(n-1)\eta_1}{n-1} - \frac{\sin(n+1)\eta_1}{n+1} \right\} + \frac{i\beta^2\Omega}{2} \left\{ \frac{\sin(n-2)\eta_1}{(n-2)(n-1)} - \frac{2\sin n\eta_1}{(n-1)(n+1)} + \right. \right. \\
&\quad + \frac{\sin(n+2)\eta_1}{(n+1)(n+2)} + \left. \left. \tau \left\{ \frac{\sin(n-3)\eta_1}{(n-3)(n-1)} - \frac{\sin(n-1)\eta_1}{(n-1)^2} - \frac{\sin(n+1)\eta_1}{(n+1)^2} + \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{\sin(n+3)\eta_1}{(n+1)(n+3)} \right\} \right] + O(n^{-3}), \quad (4,4)
\end{aligned}$$

and

$$\begin{aligned}
T_n(\beta^2\Omega, \eta_1) &= \int_0^{\eta_1} e^{i\beta^2\Omega \cos \eta} \operatorname{se}_n(\eta) \cos \eta \sin \eta d\eta = \\
&= \frac{1}{4} \sum_{r=1}^{\infty} B_r^{(n)} \{ R_{r-2}(\beta^2\Omega, \eta_1) - R_{r+2}(\beta^2\Omega, \eta_1) \} = \quad (4,5) \\
&= \frac{A}{4} \left[\left\{ \frac{\sin(n-2)\eta_1}{n-2} - \frac{\sin(n+2)\eta_1}{n+2} \right\} + \frac{i\beta^2\Omega}{2} \left\{ \frac{\sin(n-3)\eta_1}{(n-3)(n-2)} - \frac{\sin(n-1)\eta_1}{(n-2)(n-1)} - \right. \right. \\
&\quad - \frac{\sin(n+1)\eta_1}{(n+1)(n+2)} + \frac{\sin(n+3)\eta_1}{(n+2)(n+3)} \left. \right\} + \tau \left\{ \frac{\sin(n-4)\eta_1}{(n-4)(n-1)} - \frac{2\sin n\eta_1}{(n-1)(n+1)} + \frac{\sin(n+4)\eta_1}{(n+1)(n+4)} \right\}
\end{aligned}$$

One needs also

$$\frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} = -\frac{1}{n} - \frac{4\tau}{(n-1)n(n+1)} - \frac{(5n^2-11)4\tau^2}{(n-2)(n-1)^2n(n+1)^2(n+2)} \dots, \quad (4,6)$$

which result is obtained by means of section 6 of the report R 53 Int 4 and the well-known series for $J_n(x)$ and $Y_n(x)$.

From (4,6) and the expansions of R 53 Int 5 (3,5) it follows that

$$\begin{aligned} \{se_n(\eta_1) \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} + \sum_{m=1}^{\infty} B_m^{(n)} \frac{\sin m\eta_1}{m}\} = \tau \left\{ \frac{2 \sin(n-2)\eta_1}{(n-2)(n-1)n} - \right. \\ \left. - \frac{4 \sin n\eta_1}{(n-1)n(n+1)} + \frac{2 \sin(n+2)\eta_1}{n(n+1)(n+2)} \right\} + \\ + \tau^2 \left\{ \frac{2 \sin(n-4)\eta_1}{(n-4)(n-2)(n-1)n} - \frac{4 \sin(n-2)\eta_1}{(n-1)^2n(n+1)} + \frac{4 \sin(n+2)\eta_1}{(n-1)n(n+1)^2} \right. \\ \left. - \frac{2 \sin(n+4)\eta_1}{n(n+1)(n+2)(n+4)} \right\} + O\left(\frac{\tau^5}{n}\right). \quad (4,7) \end{aligned}$$

Indeed, it is clear that

$$\begin{aligned} \mathcal{A}_n(\Omega) = \int_0^{\infty} e^{-i\Omega \cosh \xi - n\xi} d\xi = \\ = \frac{1}{n} - \frac{i\Omega}{1!2} \left(\frac{1}{n-1} + \frac{1}{n+1} \right) - \frac{\Omega^2}{2!2^2} \left(\frac{1}{n-2} + \frac{2}{n} + \frac{1}{n+2} \right) + \\ + \frac{i\Omega^3}{3!2^3} \left(\frac{1}{n-3} + \frac{3}{n-1} + \frac{3}{n+1} + \frac{1}{n+3} \right) + \dots + \frac{(-i\Omega)^N}{N!2^N} \left(\frac{1}{n-N} + \frac{N}{n-N+2} + \dots + \frac{1}{n+N} \right) + \\ + R_N(\Omega), \quad (4,8) \end{aligned}$$

where $N < n$, N is a positive integer, and

$$R_N(\Omega) = \int_0^{\infty} \sum_{l=N}^{\infty} \frac{(-i\Omega \cosh \xi)^l}{l!} e^{-n\xi} d\xi, \quad (4,9)$$

because

$$\begin{aligned} \mathcal{A}_n(\Omega) = \int_0^{\infty} \sum_{l=0}^{\infty} \frac{(-i\Omega \cosh \xi)^l}{l!} e^{-n\xi} d\xi = \\ = \sum_{l=0}^N \frac{(i\Omega)^l}{l!} \int_0^{\infty} e^{-n\xi} (\cosh \xi)^l d\xi + R_N(\Omega), \end{aligned}$$

and evaluating the integrals in the last line yields readily formula (4,7). Again it may be noticed that all these expressions diverge when $N \rightarrow \infty$.

Finally it will be proved that

$$\mathcal{A}_n(\beta, \Omega) = (-1)^{n+1} \left\{ \sum_{m=1}^{\infty} B_m^{(n)} \mathcal{A}_m(\Omega) - O(\tau^{\frac{1}{2}n} \cdot (\frac{n}{2})^{-\frac{n}{2}} - 1 \cdot \frac{1}{\Omega}) \right\}. \quad (4,10)$$

Therefore one needs (5,5) and (5,6) of R 53, Int 4,7. First is to be shown

$$\mathcal{J}^{(n)} = \frac{n!(n-1)! \tau^{-n}}{4} + \dots, \quad (4,11)$$

In this order only the case of even n is dealt with. The case of odd n is similar.

Use is made of some formula's of R 53, Int 2 (9,2):

$$T_{2r}^{(2n)} = -\frac{1}{2}(-\tau)^r \frac{d}{dr} \left\{ (-\tau)^{-r} B_{2r}^{(2n)} \right\}, \quad (4,12)$$

and

$$\mathcal{J}^{(2n)} = \frac{b_{2n} T_0^{(2n)} - 2q T_2^{(2n)}}{2q B_2^{(2n)}} = \frac{b_{2n} T_0^{(2n)} - 8\tau T_2^{(2n)}}{8\tau B_2^{(2n)}}. \quad (4,13)$$

Furthermore, $T_{2r}^{(2n)}$ is the solution of the recurrence-relations of $T_m^{(2n)}$ that becomes small for large values of r . Hence, one has only to substitute the expressions (see R 53, Int 5, (3,5).

$$B_{2r}^{(2n)} = \frac{(-1)^{r-n} (2n)! \tau^{r-n}}{(r-n)!(r+n)!} + \dots,$$

and

$$T_{2r}^{(2n)} = -\frac{1}{2} (-\tau)^{r-n} (2n)! \frac{\Psi(r-n) + \Psi(r+n)}{(r-n)!(r+n)!}$$

into (4,12) and (4,13), and (4,10) results.

By means of relation (3,6) of R 53, Int 4 it is shown that

$$|I_m'(\Omega)| < (n-1)! 2^{+n+1} \Omega^n.$$

So the largest term in (5,5) of Int 4 is

$$T_0^{(n)} H_0^*(\Omega)$$

that has the order $\tau^{-\frac{1}{2}n} n! [\frac{1}{2}n]!$, where $[\frac{1}{2}n]$ denotes the greatest integer contained in $\frac{1}{2}n$.

Also,

$$\left| \frac{K_n(\beta, \Omega)}{Se_n'(0)} \right| < K_1 \frac{\tau^{\frac{1}{2}n}}{\Omega} (\frac{1}{2}n)^{-\frac{1}{2}n+1},$$

where K_1 is some constant independent of n , τ and Ω , what yields the proof of (4,9).

If at last

$$\Lambda_N(\Omega) = \sum_{h=-N+1}^{h=N-1} \frac{\theta_h}{n+h} + R_N(\Omega), \quad (4,14)$$

and one sees that except for (1,2), all series, mentioned in section 1, can be built up by the parts described above, and the method of section 3 is available to produce the results correct in 5 decimals using only the mentioned first two terms of the expansions.

For $N = 9$ the highest value of N taken in the computations, the functions θ_h are tabulated below

$$\begin{aligned} \theta_0 &= 1 - \frac{\Omega^2}{4} + \frac{\Omega^4}{64} - \frac{\Omega^6}{2304} + \frac{\Omega^8}{147456}, \\ \theta_1 = \theta_{-1} &= i \left\{ -\frac{\Omega}{2} + \frac{\Omega^3}{16} - \frac{\Omega^5}{384} + \frac{\Omega^7}{18432} \right\}, \\ \theta_2 = \theta_{-2} &= -\frac{\Omega^2}{8} + \frac{\Omega^4}{96} - \frac{\Omega^6}{3072} + \frac{\Omega^8}{184320}, \\ \theta_3 = \theta_{-3} &= i \left\{ \frac{\Omega^3}{48} - \frac{\Omega^5}{768} + \frac{\Omega^7}{30720} \right\}, \\ \theta_4 = \theta_{-4} &= \frac{\Omega^4}{384} - \frac{\Omega^6}{7680} + \frac{\Omega^8}{368640}, \\ \theta_5 = \theta_{-5} &= i \left\{ \frac{\Omega^5}{3840} + \frac{\Omega^7}{92160} \right\}, \\ \theta_6 = \theta_{-6} &= -\frac{\Omega^6}{46080} + \frac{\Omega^8}{1290240}, \\ \theta_7 = \theta_{-7} &= i \frac{\Omega^7}{645120}, \\ \theta_8 = \theta_{-8} &= \frac{\Omega^8}{10321920}. \end{aligned} \quad (4,15)$$

Now $G_n(\beta, \Omega, \eta_1)$ has to be treated, defined by

$$\begin{aligned} G_n(\beta, \Omega, \eta_1) &= (-1)^n \operatorname{se}'_n(0) \int_0^\infty e^{-i\Omega \cosh \xi} \left\{ \operatorname{se}_n(\eta_1) \frac{\operatorname{Ne}_n^{(2)}(\xi)}{\operatorname{Ne}_n^{(2)}(0)} + \right. \\ &+ \sum_{m=1}^\infty B_m^{(n)} \frac{e^{-m\xi} \sin m\eta_1}{m} d\xi = \operatorname{se}_n(\eta_1) \Lambda_n(\beta, \Omega) + \\ &+ (-1)^n \operatorname{se}'_n(0) \sum_{m=1}^\infty B_m^{(n)} \frac{\sin m\eta_1}{m} \Lambda_m(\Omega). \end{aligned} \quad (4,16)$$

Substituting (4,9), (4,14) and

$$\operatorname{se}'_n(0) = \sum_{m=1}^\infty m B_m^{(n)},$$

one readily gets

$$G_n(\beta, \Omega, \eta_1) = (-1)^n \left\{ \sum_{k=1}^{\infty} B_h^{(n)} c_h^n(\eta_1) \cdot \sum_{k=-h+1}^{h-1} \frac{\theta_k}{n+k} \right\}, \quad (4,17)$$

where

$$c_h^n(\eta_1) = \sum_{m=1}^{\infty} m B_m^{(n)} \left\{ \frac{\sin h \eta_1}{h} - \frac{\sin m \eta_1}{m} \right\}. \quad (4,18)$$

Because of the poor convergence of the series (4,14) it is necessary to take in (4,17) n equal to 8 and more. Then one has only to use the $B_n^{(n)}$, $B_{n-2}^{(n)}$ and $B_{n+2}^{(n)}$ multiplied with enough terms of the series for $\Lambda_m(\Omega)$ in order to get out results correct in 5 decimals when $T \leq \frac{1}{4}$.

$$5. \quad \sum_{n=2}^{\infty} G_n(\beta, \Omega, \eta_1).$$

$$\theta_0 \cdot q/4 \times$$

$$\left\{ \operatorname{Im} [G(\eta_1, 4) \{ e^{2i\eta_1} + 2 + e^{-2i\eta_1} \} - G(\eta_1, 2) \{ 1 + e^{2i\eta_1} \} - \right. \\ \left. - G(\eta_1, 6) \{ 1 + e^{-2i\eta_1} \} \right\} + 2 \left\{ \bar{S} \binom{9}{2}(\eta_1) - S \binom{11}{2}(\eta_1) \right\} \right\}$$

$$- q/4 \frac{\theta_1}{2} \times$$

$$\left\{ \operatorname{Im} [G(\eta_1, 5) \{ 4e^{i\eta_1} - e^{i\eta_1} - e^{-3i\eta_1} \} + G(\eta_1, 3) \{ 4e^{-i\eta_1} - e^{i\eta_1} - e^{3i\eta_1} \} + \right. \\ \left. + G(\eta_1, 7) \{ e^{-3i\eta_1} - 3e^{-i\eta_1} \} + G(\eta_1, 1) \{ e^{3i\eta_1} - 3e^{i\eta_1} \} \right\} + \\ \left. + 8 \cos \eta_1 \left\{ -\bar{S} \binom{10}{2}(\eta_1) + \bar{S} \binom{12}{2}(\eta_1) \right\} \right\}$$

$$- q/4 \frac{\theta_2}{3} \times$$

$$\left\{ \operatorname{Im} [-6 G(\eta_1, 4) + G(\eta_1, 6) \{ 3 + 2e^{-2i\eta_1} - e^{-4i\eta_1} \} + \right. \\ \left. + G(\eta_1, +2) \{ 3 + 2e^{2i\eta_1} - e^{4i\eta_1} \} + G(\eta_1, 8) \{ e^{-4i\eta_1} - 2e^{-2i\eta_1} \} + \right. \\ \left. + G(\eta_1, 0) \{ e^{4i\eta_1} - 2e^{2i\eta_1} \} \right\}$$

$$+ q/4 \frac{\theta_3}{12} \times$$

$$\left\{ \operatorname{Im} [G(\eta_1, 5) \{ 6e^{-i\eta_1} - 2e^{-3i\eta_1} \} + G(\eta_1, 3) \{ 6e^{i\eta_1} - 2e^{3i\eta_1} \} + \right. \\ \left. + G(\eta_1, 7) \{ -6e^{-i\eta_1} - 3e^{-3i\eta_1} + 3e^{-5i\eta_1} \} \right. \\ \left. + G(\eta_1, 9) \{ +5e^{-3i\eta_1} - 3e^{-5i\eta_1} \} + G(\eta_1, -1) \{ 5e^{3i\eta_1} - 3e^{5i\eta_1} \} + \right. \\ \left. + G(\eta_1, 1) \{ -6e^{i\eta_1} - 3e^{3i\eta_1} + 3e^{5i\eta_1} \} \right\}$$

$$\begin{aligned}
& + \frac{q}{4} \frac{\theta_4}{30} x \\
& \left\{ \operatorname{Im}[G(\eta_1, -2)] \{-6 e^{6i\eta_1} + 9 e^{4i\eta_1}\} + G(\eta_1, 0) \{-10 e^{2i\eta_1} - 4 e^{4i\eta_1} + 6 e^{6i\eta_1}\} + \right. \\
& \quad + G(\eta_1, +2) \{10 e^{2i\eta_1} - 5 e^{4i\eta_1}\} + G(\eta_1, 10) \{-6 e^{-6i\eta_1} + 9 e^{-4i\eta_1}\} + \\
& \quad + G(\eta_1, 8) \{-10 e^{-2i\eta_1} - 4 e^{-4i\eta_1} + 6 e^{-6i\eta_1}\} + \\
& \quad \left. + G(\eta_1, 6) \{10 e^{-2i\eta_1} - 5 e^{-4i\eta_1}\} \right\} \\
& - \frac{q}{4} \frac{\theta_5}{60} x \\
& \left\{ \operatorname{Im}[G(\eta_1, -3)] \{-14 e^{5i\eta_1} + 10 e^{7i\eta_1}\} + \right. \\
& \quad + G(\eta_1, -1) \{15 e^{3i\eta_1} + 5 e^{5i\eta_1} - 10 e^{7i\eta_1}\} + \\
& \quad + G(\eta_1, 1) \{-15 e^{3i\eta_1} + 9 e^{5i\eta_1}\} + \\
& \quad + G(\eta_1, 11) \{-14 e^{-5i\eta_1} + 10 e^{-7i\eta_1}\} + \\
& \quad + G(\eta_1, 9) \{15 e^{-3i\eta_1} + 5 e^{-5i\eta_1} - 10 e^{-7i\eta_1}\} + \\
& \quad \left. + G(\eta_1, 7) \{-15 e^{-3i\eta_1} + 9 e^{-5i\eta_1}\} \right\} \\
& - \frac{q}{4} \frac{\theta_6}{105} x \\
& \left\{ \operatorname{Im}[G(\eta_1, -4)] \{-20 e^{6i\eta_1} + 15 e^{8i\eta_1}\} + \right. \\
& \quad + G(\eta_1, -2) \{21 e^{4i\eta_1} + 6 e^{6i\eta_1} - 15 e^{8i\eta_1}\} + \\
& \quad + G(\eta_1, 0) \{-21 e^{4i\eta_1} + 14 e^{6i\eta_1}\} + \\
& \quad + G(\eta_1, 12) \{-20 e^{-6i\eta_1} + 15 e^{-8i\eta_1}\} + \\
& \quad + G(\eta_1, 10) \{21 e^{-4i\eta_1} + 6 e^{-6i\eta_1} - 15 e^{-8i\eta_1}\} + \\
& \quad \left. + G(\eta_1, 8) \{-21 e^{-4i\eta_1} + 14 e^{-6i\eta_1}\} \right\} \\
& \frac{q}{4} \frac{\theta_7}{168} x \\
& \left\{ \operatorname{Im}[G(\eta_1, -5)] \{+27 e^{7i\eta_1} - 21 e^{9i\eta_1}\} + \right. \\
& \quad + G(\eta_1, -3) \{-28 e^{5i\eta_1} - 7 e^{7i\eta_1} + 21 e^{9i\eta_1}\} + \\
& \quad + G(\eta_1, -1) \{28 e^{5i\eta_1} - 20 e^{7i\eta_1}\} + \\
& \quad + G(\eta_1, 13) \{27 e^{-7i\eta_1} - 21 e^{-9i\eta_1}\} + \\
& \quad + G(\eta_1, 11) \{-28 e^{-5i\eta_1} - 7 e^{-7i\eta_1} + 21 e^{-9i\eta_1}\} + \\
& \quad \left. + G(\eta_1, 9) \{28 e^{-5i\eta_1} - 20 e^{-7i\eta_1}\} \right\}
\end{aligned}$$

$$\begin{aligned}
& q/4 \frac{\theta_8}{252} x \\
& \left\{ \operatorname{Im}[G(\eta_1, -6)] \{ 35 e^{8i\eta_1} - 28 e^{10i\eta_1} \} + \right. \\
& \quad + G(\eta_1, -4) \{ -36 e^{6i\eta_1} - 8 e^{8i\eta_1} + 28 e^{10i\eta_1} \} + \\
& \quad + G(\eta_1, -2) \{ 36 e^{6i\eta_1} - 27 e^{8i\eta_1} \} + \\
& \quad + G(\eta_1, 14) \{ 35 e^{-8i\eta_1} - 28 e^{-10i\eta_1} \} + \\
& \quad + G(\eta_1, 12) \{ -36 e^{-6i\eta_1} - 8 e^{-8i\eta_1} + 28 e^{-10i\eta_1} \} + \\
& \quad \left. + G(\eta_1, 10) \{ 36 e^{-6i\eta_1} - 27 e^{-8i\eta_1} \} \right\}
\end{aligned}$$

$$6. -2 \sum_{n=1}^{\infty} \left\{ \operatorname{se}_n(\eta_1) \frac{N_n^{(2)}(0)}{N_n^{(2)'}(0)} + \sum_{m=1}^{\infty} B_m^{(n)} \frac{\sin m\eta_1}{m} \right\} S_n(\beta, \alpha, \eta_1).$$

$$\begin{aligned}
& q/4 A \left\{ -\frac{1}{3} \left[\operatorname{Re} \{ e^{3i\eta_1} F(2\eta_1, -4) - 3 e^{i\eta_1} F(2\eta_1, -2) + 3 e^{-i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \quad \left. \left. - e^{-3i\eta_1} F(2\eta_1, 2) \} - \frac{1}{60} \cos 3\eta_1 \right] \right. \\
& + \left[\operatorname{Re} \{ e^{i\eta_1} F(2\eta_1, -4) - e^{-3i\eta_1} F(2\eta_1, 0) \} - \right. \\
& \quad \left. - 4 \cos \eta_1 \left\{ C_2^{(6)}(2, \eta_1) + \frac{119}{288} - \frac{\pi^2}{24} \right\} - 4 \sin \eta_1 S_2^{(6)}(2, \eta_1) \right] \\
& + \left[\operatorname{Re} \{ e^{3i\eta_1} F(2\eta_1, -2) - 4 e^{i\eta_1} F(2\eta_1, 0) + 3 e^{-i\eta_1} F(2\eta_1, 2) \} + \right. \\
& \quad \left. + 4 \cos \eta_1 \left\{ C_2^{(10)}(2, \eta_1) + \frac{5899}{14400} - \frac{\pi^2}{24} \right\} + 4 \sin \eta_1 S_2^{(10)}(2, \eta_1) \right] \\
& + \left[\operatorname{Re} \{ 3 e^{i\eta_1} F(2\eta_1, -2) + e^{-3i\eta_1} F(2\eta_1, 2) - 4 e^{-i\eta_1} F(2\eta_1, 0) - \right. \\
& \quad \left. - 4 \cos \eta_1 \left\{ C_2^{(6)}(2, \eta_1) - \frac{\pi^2}{24} + \frac{589}{1440} \right\} + 4 \sin \eta_1 S_2^{(6)}(2, \eta_1) \right] \\
& - \left[\operatorname{Re} \{ e^{3i\eta_1} F(2\eta_1, 0) - e^{-i\eta_1} F(2\eta_1, 4) \} - \right. \\
& \quad \left. - 4 \cos \eta_1 \left\{ C_2^{(10)}(2, \eta_1) - \frac{\pi^2}{24} + \frac{5929}{14400} \right\} + 4 \sin \eta_1 S_2^{(10)}(2, \eta_1) \right] \\
& + \frac{1}{3} \left[\operatorname{Re} \{ e^{3i\eta_1} F(2\eta_1, -2) - 3 e^{i\eta_1} F(2\eta_1, 0) + \right. \\
& \quad \left. + 3 e^{-i\eta_1} F(2\eta_1, 2) - e^{-3i\eta_1} F(2\eta_1, 4) \} - \frac{1}{120} \cos 3\eta_1 \right] \Big\}
\end{aligned}$$

$$+ i \frac{q}{4} \beta^2 \Omega A$$

$$\left\{ \begin{aligned} & \frac{1}{12} \left[\operatorname{Re} \left\{ e^{4i\eta_1} F(2\eta_1, -4) - 4 e^{2i\eta_1} F(2\eta_1, -2) + 6 F(2\eta_1, 0) - \right. \right. \\ & \quad \left. \left. - 4 e^{-2i\eta_1} F(2\eta_1, +2) + e^{-4i\eta_1} F(2\eta_1, 4) \right\} - \frac{1}{120} \cos 4\eta_1 \right] \\ & - \frac{1}{3} \left[\operatorname{Re} \left\{ 2 e^{2i\eta_1} F(2\eta_1, -4) + 3 F(2\eta_1, -2) - 6 e^{-2i\eta_1} F(2\eta_1, 0) + \right. \right. \\ & \quad \left. \left. + e^{-4i\eta_1} F(2\eta_1, 2) \right\} - 12 C_2^{(6)}(2, \eta_1) + \cos 2\eta_1 \left(\frac{\pi^2}{2} - \frac{593}{120} \right) \right] \\ & + \frac{1}{4} \left[\operatorname{Re} \left\{ -5 F(2\eta_1, -4) + 4 e^{-2i\eta_1} F(2\eta_1, -2) + e^{-4i\eta_1} F(2\eta_1, 0) \right\} \right. \\ & \quad \left. + 4 C_2^{(4)}(2, \eta_1) + 8 \cos 2\eta_1 \cdot C_2^{(6)}(2, \eta_1) + 8 \sin 2\eta_1 \cdot S_2^{(6)}(2, \eta_1) - \right. \\ & \quad \left. - \frac{\pi^2}{2} + \frac{355}{72} \right] \\ & - \frac{1}{3} \left[\operatorname{Re} \left\{ e^{4i\eta_1} F(2\eta_1, -2) - 6 e^{2i\eta_1} F(2\eta_1, 0) + 3 F(2\eta_1, 2) + 2 e^{-2i\eta_1} F(2\eta_1, 4) \right\} \right. \\ & \quad \left. + 12 C_2^{(10)}(2, \eta_1) - \cos 2\eta_1 \left(\frac{\pi^2}{2} - \frac{5919}{1200} \right) \right] \\ & + \left[\operatorname{Re} \left\{ -2 e^{2i\eta_1} F(2\eta_1, -2) + 4 F(2\eta_1, 0) - 2 e^{-2i\eta_1} F(2\eta_1, 2) \right\} + \right. \\ & \quad \left. + 2 \cos 2\eta_1 \left(\frac{\cos 8\eta_1}{8^2} + \frac{\cos 10\eta_1}{10^2} \right) \right. \\ & \quad \left. - 2 \sin 2\eta_1 \left\{ 2 S_2(2, \eta_1) - \frac{\sin 2\eta_1}{2} - \frac{\sin 4\eta_1}{8} - \frac{\sin 6\eta_1}{18} - \right. \right. \\ & \quad \left. \left. - \frac{\sin 8\eta_1}{8^2} - \frac{\sin 10\eta_1}{10^2} \right\} - \frac{1}{800} \right] \\ & + \frac{1}{4} \left[\operatorname{Re} \left\{ e^{4i\eta_1} F(2\eta_1, 0) + 4 e^{2i\eta_1} F(2\eta_1, 2) - 5 F(2\eta_1, 4) \right\} - \right. \\ & \quad \left. - 4 C_2^{(12)}(2, \eta_1) - 8 \cos 2\eta_1 \cdot C_2^{(10)}(2, \eta_1) + \right. \\ & \quad \left. + 8 \sin 2\eta_1 \cdot S_2^{(10)}(2, \eta_1) + \frac{\pi^2}{2} - \frac{17767}{3600} \right] \Bigg\}. \end{aligned}$$

$$+ q^2/16 A x$$

$$\left\{ - \left[\operatorname{Re} \left\{ e^{i\eta_1} F(2\eta_1, -4) - 2 e^{-i\eta_1} F(2\eta_1, -2) + e^{-3i\eta_1} F(2\eta_1, 0) \right\} - \right. \right. \\ \left. \left. - 8 \cos \eta_1 \left\{ C_3^{(6)}(2, \eta_1) - \frac{1}{8} \zeta(3) + \frac{65}{432} \right\} - 8 \sin \eta_1 \cdot S_3^{(6)}(2, \eta_1) \right] \right\}$$

$$\begin{aligned}
& + \frac{1}{6} \left[\operatorname{Re} \left\{ e^{i\eta_1} F(2\eta_1, -6) - 6 e^{-i\eta_1} F(2\eta_1, -4) + 3 e^{-3i\eta_1} F(2\eta_1, -2) + \right. \right. \\
& \quad + 2 e^{-5i\eta_1} F(2\eta_1, 0) \left. \right\} + 12 \cos 3\eta_1 \cdot C_2^{(6)}(2, \eta_1) + \\
& \quad + 12 \sin 3\eta_1 \cdot S_2^{(6)}(2, \eta_1) - \cos \eta_1 \left(\frac{\pi^2}{2} - \frac{59}{12} \right) \left. \right] \\
& + \frac{1}{60} \left[\operatorname{Re} \left\{ 4 e^{5i\eta_1} F(2\eta_1, -4) - 15 e^{3i\eta_1} F(2\eta_1, -2) + 20 e^{i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \quad - 10 e^{-i\eta_1} F(2\eta_1, 2) + e^{-5i\eta_1} F(2\eta_1, 6) \left. \right\} - \frac{31}{840} \cos 5\eta_1 \left. \right] \\
& - \frac{1}{18} \left[\operatorname{Re} \left\{ 2 e^{3i\eta_1} F(2\eta_1, -4) - 9 e^{i\eta_1} F(2\eta_1, -2) + 18 e^{-i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \quad - 11 e^{-3i\eta_1} F(2\eta_1, 2) \left. \right\} - 12 \cos 3\eta_1 \left\{ C_2^{(10)}(2, \eta_1) - \frac{\pi^2}{24} + \frac{5939}{14400} \right\} - \\
& \quad - 12 \sin 3\eta_1 \cdot S_2^{(10)}(2, \eta_1) \left. \right] \\
& + \frac{1}{2} \left[\operatorname{Re} \left\{ 7 e^{i\eta_1} F(2\eta_1, -2) - 8 e^{-i\eta_1} F(2\eta_1, 0) + e^{-3i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \quad + 48 \cos \eta_1 \left\{ \frac{1}{3} C_3^{(6)}(2, \eta_1) - \frac{1}{4} C_2^{(6)}(2, \eta_1) - \frac{1}{24} \zeta(3) + \frac{\pi^2}{96} - \frac{2737}{51840} \right\} - \\
& \quad - 48 \sin \eta_1 \left\{ \frac{1}{3} S_3^{(6)}(2, \eta_1) - \frac{1}{4} S_2^{(6)}(2, \eta_1) \right\} \left. \right] \\
& - \frac{1}{12} \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, -6) + 12 e^{-i\eta_1} F(2\eta_1, -2) - 16 e^{-3i\eta_1} F(2\eta_1, 0) + \right. \right. \\
& \quad + 3 e^{-5i\eta_1} F(2\eta_1, 2) \left. \right\} - 24 \cos \eta_1 \cdot C_2^{(6)}(2, \eta_1) - \\
& \quad - 24 \sin \eta_1 \cdot S_2^{(6)}(2, \eta_1) - \cos 3\eta_1 \left(\frac{1189}{120} - \pi^2 \right) \left. \right] \\
& - \frac{1}{12} \left[\operatorname{Re} \left\{ 3 e^{5i\eta_1} F(2\eta_1, -2) - 16 e^{3i\eta_1} F(2\eta_1, 0) + 12 e^{i\eta_1} F(2\eta_1, 2) + \right. \right. \\
& \quad + e^{-3i\eta_1} F(2\eta_1, 6) \left. \right\} + 24 \cos \eta_1 \cdot C_2^{(10)}(2, \eta_1) - \\
& \quad - 24 \sin \eta_1 \cdot S_2^{(10)}(2, \eta_1) - \cos 3\eta_1 \left(\pi^2 - \frac{20709}{2100} \right) \left. \right] \\
& + \frac{1}{2} \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, -2) - 8 e^{i\eta_1} F(2\eta_1, 0) + 7 e^{-i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \quad + 48 \cos \eta_1 \left\{ \frac{1}{3} C_3^{(10)}(2, \eta_1) + \frac{1}{4} C_2^{(10)}(2, \eta_1) - \frac{1}{24} \zeta(3) - \frac{\pi^2}{96} + \right. \\
& \quad \left. \left. + \frac{792413}{5184000} \right\} + \right. \\
& \quad \left. + 48 \sin \eta_1 \left\{ \frac{1}{3} S_3^{(10)}(2, \eta_1) + \frac{1}{4} S_2^{(10)}(2, \eta_1) \right\} \right] \\
& - \frac{1}{18} \left[\operatorname{Re} \left\{ -11 e^{3i\eta_1} F(2\eta_1, -2) + 18 e^{i\eta_1} F(2\eta_1, 0) - 9 e^{-i\eta_1} F(2\eta_1, 2) + 2 e^{-3i\eta_1} F(2\eta_1, 4) \right\} + \right. \\
& \quad + 12 \cos 3\eta_1 \left\{ C_2^{(6)}(2, \eta_1) - \frac{\pi^2}{24} + \frac{59}{1440} \right\} - 12 \sin 3\eta_1 \cdot S_2^{(6)}(2, \eta_1) \left. \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{60} \left[\operatorname{Re} \left\{ e^{5i\eta_1} F(2\eta_1, -6) - 10 e^{i\eta_1} F(2\eta_1, -2) + 20 e^{-i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \quad \left. \left. - 15 e^{-3i\eta_1} F(2\eta_1, 2) + 4 e^{-5i\eta_1} F(2\eta_1, 4) \right\} - \frac{7 \cos 5\eta_1}{120} \right] \\
& + \frac{1}{6} \left[\operatorname{Re} \left\{ 2 e^{5i\eta_1} F(2\eta_1, 0) + 3 e^{3i\eta_1} F(2\eta_1, 2) - 6 e^{i\eta_1} F(2\eta_1, 4) + e^{-i\eta_1} F(2\eta_1, 6) \right\} - \right. \\
& \quad \left. - 12 \cos 3\eta_1 \cdot C_2^{(10)}(2, \eta_1) - \cos \eta_1 \left(\frac{41463}{8400} - \frac{\pi^2}{2} \right) + 12 \sin 3\eta_1 \cdot S_2^{(10)}(2, \eta_1) \right] \\
& - \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, 0) - 2 e^{i\eta_1} F(2\eta_1, 2) + e^{-i\eta_1} F(2\eta_1, 4) \right\} - \right. \\
& \quad \left. - 8 \cos \eta_1 \left\{ C_3^{(10)}(2, \eta_1) + \frac{259703}{1728000} - \frac{1}{8} \zeta(3) \right\} + 8 \sin \eta_1 \cdot S_3^{(10)}(2, \eta_1) \right] \\
& - \frac{1}{60} \left[\operatorname{Re} \left\{ e^{5i\eta_1} F(2\eta_1, -8) - 10 e^{i\eta_1} F(2\eta_1, -4) + 20 e^{-i\eta_1} F(2\eta_1, -2) - \right. \right. \\
& \quad \left. \left. - 15 e^{-3i\eta_1} F(2\eta_1, 0) + 4 e^{-5i\eta_1} F(2\eta_1, 2) \right\} - \frac{9 \cos 5\eta_1}{40} \right] \\
& + \left[\operatorname{Re} \left\{ -2 e^{i\eta_1} F(2\eta_1, -2) + 4 e^{-i\eta_1} F(2\eta_1, 0) - 2 e^{-3i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \quad + 2 \cos \eta_1 \cdot C_2^{(6)}(2, \eta_1) - 2 \cos 3\eta_1 \left\{ C_2^{(10)}(2, \eta_1) + \frac{1}{1600} \right\} - \\
& \quad \left. - 2 \sin \eta_1 \cdot S_2^{(6)}(2, \eta_1) - 2 \sin 3\eta_1 \cdot S_2^{(10)}(2, \eta_1) \right] \\
& - \frac{1}{2} \left[\operatorname{Re} \left\{ e^{5i\eta_1} F(2\eta_1, -2) - 8 e^{3i\eta_1} F(2\eta_1, 0) + 7 e^{i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \quad + 48 \cos \eta_1 \left\{ \frac{1}{3} C_3^{(10)}(2, \eta_1) + \frac{1}{4} C_2^{(10)}(2, \eta_1) - \frac{1}{24} \zeta(3) - \frac{\pi^2}{96} + \right. \\
& \quad \left. \left. + \frac{792413}{5184000} \right\} - \right. \\
& \quad \left. - 48 \sin \eta_1 \left\{ \frac{1}{3} S_3^{(10)}(2, \eta_1) + \frac{1}{4} S_2^{(10)}(2, \eta_1) \right\} \right] \\
& + \frac{1}{36} \left[\operatorname{Re} \left\{ 9 e^{5i\eta_1} F(2\eta_1, 0) + 8 e^{3i\eta_1} F(2\eta_1, 2) - 18 e^{i\eta_1} F(2\eta_1, 4) + \right. \right. \\
& \quad \left. \left. + e^{-3i\eta_1} F(2\eta_1, 8) \right\} - \right. \\
& \quad \left. - 48 \cos 3\eta_1 \left\{ C_2^{(10)}(2, \eta_1) + \frac{55289}{134400} - \frac{\pi^2}{24} \right\} + 48 \sin 3\eta_1 S_2^{(10)}(2, \eta_1) \right] \\
& + \frac{1}{36} \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, -8) - 18 e^{-i\eta_1} F(2\eta_1, -4) + 8 e^{-3i\eta_1} F(2\eta_1, -2) + \right. \right. \\
& \quad \left. \left. + 9 e^{-5i\eta_1} F(2\eta_1, 0) \right\} + \right. \\
& \quad \left. + 48 \cos 3\eta_1 \left\{ C_2^{(6)}(2, \eta_1) - \frac{\pi^2}{24} + \frac{469}{1152} \right\} + 48 \sin 3\eta_1 \cdot S_2^{(6)}(2, \eta_1) \right] \\
& - \frac{1}{2} \left[\operatorname{Re} \left\{ 7 e^{-i\eta_1} F(2\eta_1, -2) - 8 e^{-3i\eta_1} F(2\eta_1, 0) + e^{-5i\eta_1} F(2\eta_1, 2) \right\} + 48 \cos \eta_1 \left\{ \frac{1}{3} C_3^{(6)}(2, \eta_1) - \right. \right. \\
& \quad \left. \left. - \frac{1}{4} C_2^{(6)}(2, \eta_1) - \frac{1}{24} \zeta(3) + \frac{\pi^2}{96} - \frac{2737}{51840} \right\} + 48 \sin \eta_1 \left\{ \frac{1}{3} S_3^{(6)}(2, \eta_1) - \frac{1}{4} S_2^{(6)}(2, \eta_1) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& + \left[\operatorname{Re} \left\{ -2e^{3i\eta_1} F(2\eta_1, -2) + 4e^{i\eta_1} F(2\eta_1, 0) - 2e^{-i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \quad + 2 \cos 3\eta_1 \left\{ C_2^{(6)}(2, \eta_1) - \frac{1}{1600} \right\} - 2 \cosh \eta_1 \cdot C_2^{(10)}(2, \eta_1) - \\
& \quad \left. - 2 \sin 3\eta_1 \cdot S_2^{(6)}(2, \eta_1) - 2 \sinh \eta_1 S_2^{(10)}(2, \eta_1) \right] \\
& - \frac{1}{60} \left[\operatorname{Re} \left\{ 4e^{5i\eta_1} F(2\eta_1, -2) - 15e^{3i\eta_1} F(2\eta_1, 0) + 20e^{i\eta_1} F(2\eta_1, 2) - \right. \right. \\
& \quad \left. \left. - 10e^{-i\eta_1} F(2\eta_1, 4) + e^{-5i\eta_1} F(2\eta_1, 8) \right\} - \frac{9 \cos 5\eta_1}{560} \right] \Bigg\}
\end{aligned}$$

$$7. - 4 \sum_5^\infty n \left\{ \operatorname{se}_n(\eta_1) \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} + \sum_1^\infty B_m^{(n)} \frac{\sin m \eta_1}{m} \right\} T_n(\beta, \Omega, \eta_1).$$

$9/4 \text{ A x}$

$$\begin{aligned}
& \left\{ -\frac{1}{12} \left[\operatorname{Re} \left\{ +3e^{+4i\eta_1} F(2\eta_1, -4) - 8e^{2i\eta_1} F(2\eta_1, -2) + 6F(2\eta_1, 0) - e^{-4i\eta_1} F(2\eta_1, 4) \right\} - \right. \right. \\
& \quad \left. \left. - \frac{7}{120} \cos 4\eta_1 \right] \right. \\
& + \frac{2}{3} \left[\operatorname{Re} \left\{ e^{4i\eta_1} F(2\eta_1, -2) - 3e^{2i\eta_1} F(2\eta_1, 0) + 3F(2\eta_1, 2) - e^{-2i\eta_1} F(2\eta_1, 4) \right\} - \right. \\
& \quad \left. - \frac{1}{120} \cos 2\eta_1 \right] \\
& - \frac{1}{4} \left[\operatorname{Re} \left\{ 2e^{4i\eta_1} F(2\eta_1, 0) - 8e^{2i\eta_1} F(2\eta_1, 2) + 6F(2\eta_1, 4) \right\} + \right. \\
& \quad \left. + 8C_2^{(12)}(2, \eta_1) - \frac{\pi^2}{3} + \frac{5909}{1800} \right] \\
& + \frac{1}{2} \left[\operatorname{Re} \left\{ -3F(2\eta_1, -4) + 4e^{-2i\eta_1} F(2\eta_1, -2) - e^{-4i\eta_1} F(2\eta_1, 0) \right\} + \right. \\
& \quad \left. + 4C_2^{(4)}(2, \eta_1) + \frac{13}{8} - \frac{\pi^2}{6} \right] \\
& - \frac{2}{3} \left[\operatorname{Re} \left\{ e^{2i\eta_1} F(2\eta_1, -4) - 3F(2\eta_1, -2) + 3e^{-2i\eta_1} F(2\eta_1, 0) - e^{-4i\eta_1} F(2\eta_1, 2) \right\} - \right. \\
& \quad \left. - \frac{1}{60} \cos 2\eta_1 \right] \\
& + \frac{1}{12} \left[\operatorname{Re} \left\{ e^{4i\eta_1} F(2\eta_1, -4) - 6F(2\eta_1, 0) + 8e^{-2i\eta_1} F(2\eta_1, 2) - 3e^{-4i\eta_1} F(2\eta_1, 4) \right\} - \right. \\
& \quad \left. - \frac{1}{24} \cos 4\eta_1 \right] \Bigg\}
\end{aligned}$$

$$+ i \frac{9}{4} \beta^2 \Omega \cdot A \times$$

$$\begin{aligned} & \left\{ -\frac{1}{12} \left[\operatorname{Re} \left\{ e^{5i\eta_1} F(2\eta_1, -2) - 4 e^{3i\eta_1} F(2\eta_1, 0) + 6 e^{i\eta_1} F(2\eta_1, 2) - \right. \right. \right. \\ & \quad \left. \left. - 4 e^{-i\eta_1} F(2\eta_1, 4) + e^{-3i\eta_1} F(2\eta_1, 6) \right\} - \frac{1}{280} \cos 3\eta_1 \right] \\ & + \frac{1}{12} \left[\operatorname{Re} \left\{ e^{5i\eta_1} F(2\eta_1, 0) - 6 e^{3i\eta_1} F(2\eta_1, 2) + 3 e^{i\eta_1} F(2\eta_1, 4) + 2 e^{-i\eta_1} F(2\eta_1, 6) \right\} + \right. \\ & \quad \left. + 12 \cos \eta_1 \cdot C_2^{(12)}(2, \eta_1) - 12 \sin \eta_1 \cdot S_2^{(12)}(2, \eta_1) - \cos \eta_1 \left(\frac{\pi^2}{2} - \frac{41443}{8400} \right) \right] \\ & - \frac{1}{24} \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, -4) - 4 e^{i\eta_1} F(2\eta_1, -2) + 6 e^{-i\eta_1} F(2\eta_1, 0) - \right. \right. \\ & \quad \left. \left. - 4 e^{-3i\eta_1} F(2\eta_1, 2) + e^{-5i\eta_1} F(2\eta_1, 4) \right\} - \frac{1}{120} \cos 3\eta_1 \right] \\ & + \frac{1}{6} \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, -2) - 6 e^{i\eta_1} F(2\eta_1, 0) + 3 e^{-i\eta_1} F(2\eta_1, 2) + 2 e^{-3i\eta_1} F(2\eta_1, 4) \right\} + \right. \\ & \quad \left. + 12 \cos \eta_1 \cdot C_2^{(10)}(2, \eta_1) + 12 \sin \eta_1 \cdot S_2^{(10)}(2, \eta_1) - \cos \eta_1 \left(\frac{\pi^2}{2} - \frac{1973}{400} \right) \right] \\ & - \frac{1}{4} \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, 0) + 4 e^{i\eta_1} F(2\eta_1, 2) - 5 e^{-i\eta_1} F(2\eta_1, 4) \right\} - \right. \\ & \quad - 8 \cos \eta_1 \cdot C_2^{(10)}(2, \eta_1) + 8 \sin \eta_1 \cdot S_2^{(10)}(2, \eta_1) - 4 \cos \eta_1 \cdot C_2^{(12)}(2, \eta_1) - \\ & \quad \left. - 4 \sin \eta_1 \cdot S_2^{(12)}(2, \eta_1) - \cos \eta_1 \left(\frac{17767}{3600} - \frac{\pi^2}{2} \right) \right] \\ & - \frac{1}{4} \left[\operatorname{Re} \left\{ -5 e^{i\eta_1} F(2\eta_1, -4) + 4 e^{-i\eta_1} F(2\eta_1, -2) + e^{-3i\eta_1} F(2\eta_1, 0) \right\} + \right. \\ & \quad + 8 \cos \eta_1 \cdot C_2^{(6)}(2, \eta_1) + 8 \sin \eta_1 \cdot S_2^{(6)}(2, \eta_1) + 4 \cos \eta_1 \cdot C_2^{(4)}(2, \eta_1) - \\ & \quad \left. - 4 \sin \eta_1 \cdot S_2^{(4)}(2, \eta_1) - \cos \eta_1 \left(\frac{\pi^2}{2} - \frac{355}{72} \right) \right] \\ & + \frac{1}{6} \left[\operatorname{Re} \left\{ 2 e^{3i\eta_1} F(2\eta_1, -4) + 3 e^{i\eta_1} F(2\eta_1, -2) - 6 e^{-i\eta_1} F(2\eta_1, 0) + \right. \right. \\ & \quad \left. \left. + e^{-3i\eta_1} F(2\eta_1, 2) \right\} - \right. \\ & \quad \left. - 12 \cos \eta_1 \cdot C_2^{(6)}(2, \eta_1) + 12 \sin \eta_1 \cdot S_2^{(6)}(2, \eta_1) - \cos \eta_1 \left(\frac{593}{120} - \frac{\pi^2}{2} \right) \right] \\ & - \frac{1}{24} \left[\operatorname{Re} \left\{ e^{5i\eta_1} F(2\eta_1, -4) - 4 e^{3i\eta_1} F(2\eta_1, -2) + 6 e^{i\eta_1} F(2\eta_1, 0) - \right. \right. \\ & \quad \left. \left. - 4 e^{-i\eta_1} F(2\eta_1, 2) + e^{-3i\eta_1} F(2\eta_1, 4) \right\} - \frac{1}{120} \cos 3\eta_1 \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{12} \left[\operatorname{Re} \left\{ 2 e^{i\eta_1} F(2\eta_1, -6) + 3 e^{-i\eta_1} F(2\eta_1, -4) - 6 e^{-3i\eta_1} F(2\eta_1, -2) + \right. \right. \\
& \quad \left. \left. + e^{-5i\eta_1} F(2\eta_1, 0) \right\} - \right. \\
& \quad \left. - 12 \cos \eta_1 \cdot C_2^{(4)}(2, \eta_1) - 12 \sin \eta_1 \cdot S_2^{(4)}(2, \eta_1) - \cos \eta_1 \left(\frac{119}{24} - \frac{\pi^2}{2} \right) \right] \\
& - \frac{1}{12} \left[\operatorname{Re} \left\{ e^{3i\eta_1} F(2\eta_1, -6) - 4 e^{i\eta_1} F(2\eta_1, -4) + 6 e^{-i\eta_1} F(2\eta_1, -2) - \right. \right. \\
& \quad \left. \left. - 4 e^{-3i\eta_1} F(2\eta_1, 0) + e^{-5i\eta_1} F(2\eta_1, 2) \right\} - \frac{1}{40} \cos 5\eta_1 \right] \\
& + \frac{1}{120} \left[\operatorname{Re} \left\{ 2 e^{5i\eta_1} F(2\eta_1, -6) - 5 e^{3i\eta_1} F(2\eta_1, -4) + 10 e^{-i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \quad \left. \left. - 10 e^{-3i\eta_1} F(2\eta_1, 2) + 3 e^{-5i\eta_1} F(2\eta_1, 4) \right\} - \frac{3}{40} \cos 5\eta_1 \right] \\
& + \frac{1}{120} \left[\operatorname{Re} \left\{ 3 e^{5i\eta_1} F(2\eta_1, -4) - 10 e^{3i\eta_1} F(2\eta_1, -2) + 10 e^{i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \quad \left. \left. - 5 e^{-3i\eta_1} F(2\eta_1, 4) + 2 e^{-5i\eta_1} F(2\eta_1, 6) \right\} - \frac{9}{280} \cos 5\eta_1 \right] \Big\} \\
& + q^2/16 \cdot A \times \\
& \left\{ - \frac{1}{72} \left[\operatorname{Re} \left\{ e^{6i\eta_1} F(2\eta_1, -8) - 9 e^{2i\eta_1} F(2\eta_1, -4) + 16 F(2\eta_1, -2) - \right. \right. \right. \\
& \quad \left. \left. - 9 e^{-2i\eta_1} F(2\eta_1, 0) + e^{-6i\eta_1} F(2\eta_1, 4) \right\} - \frac{7}{30} \cos 6\eta_1 \right] \right. \\
& + \frac{1}{9} \left[\operatorname{Re} \left\{ -11 e^{2i\eta_1} F(2\eta_1, -2) + 18 F(2\eta_1, 0) - 9 e^{-2i\eta_1} F(2\eta_1, 2) - \right. \right. \\
& \quad \left. \left. + 2 e^{-4i\eta_1} F(2\eta_1, 4) \right\} + \right. \\
& \quad \left. + 12 \cos 2\eta_1 C_2^{(6)}(2, \eta_1) - 12 \sin 2\eta_1 S_2^{(6)}(2, \eta_1) - \cos 4\eta_1 \left(\frac{\pi^2}{2} - \frac{595}{120} \right) \right] \\
& - \frac{1}{6} \left[\operatorname{Re} \left\{ 2 e^{6i\eta_1} F(2\eta_1, -2) - 12 e^{4i\eta_1} F(2\eta_1, 8) + 6 e^{2i\eta_1} F(2\eta_1, 2) + 4 F(2\eta_1, 4) \right\} + \right. \\
& \quad \left. + 24 \cos 2\eta_1 C_2^{(10)}(2, \eta_1) - 24 \sin 2\eta_1 S_2^{(10)}(2, \eta_1) - \left(\pi^2 - \frac{1973}{200} \right) \right] \\
& + \frac{P}{24} \left[\operatorname{Re} \left\{ 3 e^{6i\eta_1} F(2\eta_1, 0) - 16 e^{4i\eta_1} F(2\eta_1, 2) + 12 e^{2i\eta_1} F(2\eta_1, 4) + \right. \right. \\
& \quad \left. \left. + e^{-2i\eta_1} F(2\eta_1, 8) \right\} + \right. \\
& \quad \left. + 24 \cos 2\eta_1 \cdot C_2^{(12)}(2, \eta_1) - 24 \sin 2\eta_1 S_2^{(12)}(2, \eta_1) - \cos 2\eta_1 \cdot \right. \\
& \quad \left. \left(\pi^2 - \frac{82871}{8400} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{24} \left[\operatorname{Re} \left\{ e^{2i\eta_1} F(2\eta_1, -8) + 12 e^{-2i\eta_1} F(2\eta_1, -4) - 16 e^{-4i\eta_1} F(2\eta_1, -2) + \right. \right. \\
& \qquad \qquad \qquad \left. \left. + 3 e^{-6i\eta_1} F(2\eta_1, 0) \right\} - \right. \\
& \qquad \qquad \qquad \left. - 24 \cos 2\eta_1 \cdot C_2^{(4)}(2, \eta_1) - 24 \sin 2\eta_1 S_2^{(4)}(2, \eta_1) \cos 2\eta_1 \left(\frac{241}{24} - \pi^2 \right) \right] \\
& + \frac{1}{6} \left[\operatorname{Re} \left\{ -4 F(2\eta_1, -4) - 6 e^{-2i\eta_1} F(2\eta_1, -2) + 2 e^{-4i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \qquad \qquad \qquad \left. \left. - 2 e^{-6i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \qquad \qquad \qquad \left. + 24 \cos 2\eta_1 \cdot C_2^{(6)}(2, \eta_1) + 24 \sin 2\eta_1 \cdot S_2^{(6)}(2, \eta_1) - \left(\pi^2 - \frac{593}{60} \right) \right] \\
& + \frac{1}{9} \left[\operatorname{Re} \left\{ 2 e^{4i\eta_1} F(2\eta_1, -4) - 9 e^{2i\eta_1} F(2\eta_1, -2) + 18 F(2\eta_1, 0) - \right. \right. \\
& \qquad \qquad \qquad \left. \left. - 11 e^{-2i\eta_1} F(2\eta_1, 2) \right\} - \right. \\
& \qquad \qquad \qquad \left. - 12 \cos 2\eta_1 \cdot C_2^{(10)}(2, \eta_1) - 12 \sin 2\eta_1 \cdot S_2^{(10)}(2, \eta_1) - \right. \\
& \qquad \qquad \qquad \left. - \cos 4\eta_1 \left(-\frac{\pi^2}{2} + \frac{5939}{1200} \right) \right] \\
& - \frac{1}{72} \left[\operatorname{Re} \left\{ e^{6i\eta_1} F(2\eta_1, -4) - 9 e^{2i\eta_1} F(2\eta_1, 0) + 16 F(2\eta_1, 2) - 9 e^{-2i\eta_1} F(2\eta_1, 4) + \right. \right. \\
& \qquad \qquad \qquad \left. \left. + e^{-6i\eta_1} F(2\eta_1, 8) \right\} - \frac{41}{1680} \cos 5\eta_1 \right] \\
& - \frac{1}{2} \left[\operatorname{Re} \left\{ e^{2i\eta_1} F(2\eta_1, -4) + 4 F(2\eta_1, -2) - 5 e^{-2i\eta_1} F(2\eta_1, 0) \right\} - \right. \\
& \qquad \qquad \qquad \left. - 4 \cos 2\eta_1 \cdot C_2^{(8)}(2, \eta_1) - 4 \sin 2\eta_1 S_2^{(8)}(2, \eta_1) - 3 C_2^{(6)}(2, \eta_1) - \right. \\
& \qquad \qquad \qquad \left. - \cos 2\eta_1 \left(\frac{79}{16} - \frac{\pi^2}{2} \right) \right] \\
& + \left[\operatorname{Re} \left\{ -5 e^{2i\eta_1} F(2\eta_1, -2) + 4 F(2\eta_1, 0) + 2 e^{-2i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \qquad \qquad \qquad \left. + 8 C_2^{(8)}(2, \eta_1) + 4 \cos 2\eta_1 \cdot C_2^{(6)}(2, \eta_1) - 4 \sin 2\eta_1 \cdot S_2^{(6)}(2, \eta_1) - \right. \\
& \qquad \qquad \qquad \left. - \left(\frac{\pi^2}{2} - \frac{74}{15} \right) \right] \\
& - \frac{1}{6} \left[\operatorname{Re} \left\{ 2 e^{4i\eta_1} F(2\eta_1, -2) + 3 e^{2i\eta_1} F(2\eta_1, 0) - 6 F(2\eta_1, 2) + e^{-2i\eta_1} F(2\eta_1, 4) \right\} - \right. \\
& \qquad \qquad \qquad \left. - 12 \cos 2\eta_1 \cdot C_2^{(8)}(2, \eta_1) + 12 \sin 2\eta_1 S_2^{(8)}(2, \eta_1) - \cos 2\eta_1 \left(-\frac{\pi^2}{2} + \frac{79}{16} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{36} \left[\operatorname{Re} \left\{ e^{2i\eta_1} F(2\eta_1, -8) - 18 e^{-2i\eta_1} F(2\eta_1, -4) + 8 e^{-4i\eta_1} F(2\eta_1, -2) + \right. \right. \\
& \qquad \qquad \qquad \left. \left. + 9 e^{-6i\eta_1} F(2\eta_1, 0) \right\} \right. \\
& \qquad + 48 \cos 4\eta_1 \cdot C_2^{(6)}(2, \eta_1) + 48 \sin 4\eta_1 \cdot S_2^{(6)}(2, \eta_1) - \\
& \qquad \left. - \cos 2\eta_1 \left(2\pi^2 - \frac{469}{24} \right) \right] \\
& - \frac{1}{45} \left[\operatorname{Re} \left\{ e^{4i\eta_1} F(2\eta_1, -8) + 35 e^{-2i\eta_1} F(2\eta_1, -2) - 45 e^{-4i\eta_1} F(2\eta_1, 0) + \right. \right. \\
& \qquad \qquad \qquad \left. \left. + 9 e^{-6i\eta_1} F(2\eta_1, 2) \right\} - \right. \\
& \qquad - 60 \cos 2\eta_1 \cdot C_2^{(6)}(2, \eta_1) - 60 \sin 2\eta_1 \cdot S_2^{(6)}(2, \eta_1) - \cos 4\eta_1 \left(\frac{374}{15} - \frac{5\pi^2}{2} \right) \left. \right] \\
& + \frac{1}{180} \left[\operatorname{Re} \left\{ e^{6i\eta_1} F(2\eta_1, -8) - 20 F(2\eta_1, -2) + 45 e^{-2i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \qquad \qquad \qquad \left. \left. - 36 e^{-4i\eta_1} F(2\eta_1, 2) + 10 e^{-6i\eta_1} F(2\eta_1, 4) \right\} - \frac{37}{120} \cos 6\eta_1 \right] \\
& + \frac{1}{180} \left[\operatorname{Re} \left\{ 10 e^{6i\eta_1} F(2\eta_1, -4) - 36 e^{4i\eta_1} F(2\eta_1, -2) + 45 e^{2i\eta_1} F(2\eta_1, 0) - \right. \right. \\
& \qquad \qquad \qquad \left. \left. - 20 F(2\eta_1, 2) + e^{-6i\eta_1} F(2\eta_1, 8) \right\} - \frac{167}{1680} \cos 6\eta_1 \right] \\
& - \frac{1}{45} \left[\operatorname{Re} \left\{ 9 e^{6i\eta_1} F(2\eta_1, -2) - 45 e^{4i\eta_1} F(2\eta_1, 0) + 35 e^{2i\eta_1} F(2\eta_1, 2) + \right. \right. \\
& \qquad \qquad \qquad \left. \left. + e^{-4i\eta_1} F(2\eta_1, 8) \right\} + \right. \\
& \qquad + 60 \cos 2\eta_1 \cdot C_2^{(10)}(2, \eta_1) - 60 \sin 2\eta_1 \cdot S_2^{(10)}(2, \eta_1) - \\
& \qquad \left. - \cos 4\eta_1 \left(\frac{5\pi^2}{2} - \frac{6901}{280} \right) \right] \\
& + \frac{1}{36} \left[\operatorname{Re} \left\{ 9 e^{6i\eta_1} F(2\eta_1, 0) + 8 e^{4i\eta_1} F(2\eta_1, 2) - 18 e^{2i\eta_1} F(2\eta_1, 4) + \right. \right. \\
& \qquad \qquad \qquad \left. \left. + e^{-2i\eta_1} F(2\eta_1, 8) \right\} - \right. \\
& \qquad - 48 \cos 4\eta_1 \cdot C_2^{(10)}(2, \eta_1) + 48 \sin 4\eta_1 \cdot S_2^{(10)}(2, \eta_1) - \\
& \qquad \left. - \cos 2\eta_1 \left(\frac{55289}{2800} - 2\pi^2 \right) \right] \\
& - \frac{1}{6} \left[\operatorname{Re} \left\{ e^{2i\eta_1} F(2\eta_1, -4) - 6 F(2\eta_1, -2) + 3 e^{-2i\eta_1} F(2\eta_1, 0) + \right. \right. \\
& \qquad \qquad \qquad \left. \left. + 2 e^{-4i\eta_1} F(2\eta_1, 2) \right\} + \right. \\
& \qquad + 12 \cos 2\eta_1 \cdot C_2^{(8)}(2, \eta_1) + 12 \sin 2\eta_1 \cdot S_2^{(8)}(2, \eta_1) - \cos 2\eta_1 \left(\frac{\pi^2}{2} - \frac{1183}{240} \right) \left. \right]
\end{aligned}$$

$$\begin{aligned}
& + \left[\operatorname{Re} \left\{ e^{2i\eta_1} F(2\eta_1, -2) + 4 F(2\eta_1, 0) - 5 e^{-2i\eta_1} F(2\eta_1, 2) \right\} - 8 C_2^{(8)}(2, \eta_1) - \right. \\
& \quad \left. - 4 \cos 2\eta_1 \cdot C_2^{(10)}(2, \eta_1) - 4 \sin 2\eta_1 \cdot S_2^{(10)}(2, \eta_1) - \left(\frac{5923}{1200} - \frac{\pi^2}{2} \right) \right] \\
& - \frac{1}{2} \left[\operatorname{Re} \left\{ -5 e^{2i\eta_1} F(2\eta_1, 0) + 4 F(2\eta_1, 2) + e^{-2i\eta_1} F(2\eta_1, 4) \right\} + 3 C_2^{(10)}(2, \eta_1) + \right. \\
& \quad \left. + 4 \cos \eta_1 \cdot C_2^{(8)}(2, \eta_1) - 4 \sin \eta_1 \cdot S_2^{(8)}(2, \eta_1) - \cos 2\eta_1 \left(\frac{\pi^2}{2} - \frac{5923}{1200} \right) \right]
\end{aligned}$$

$$8. \quad -4 \sum_{n=1}^{\infty} n \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} S_n(\beta, \alpha, \eta_1) S_n^*(\beta, \alpha, \eta_1).$$

$$\begin{aligned}
& + \operatorname{Re} \left\{ F(2\eta_1, 2) - e^{2i\eta_1} F(2\eta_1, 0) \right\} + 2 C_2^{(10)}(2, \eta_1) - \frac{\pi^2}{12} + \frac{5999}{7200} \\
& + \operatorname{Re} \left\{ e^{-2i\eta_1} F(2\eta_1, 2) - 2 F(2\eta_1, 0) + e^{2i\eta_1} F(2\eta_1, -2) \right\} - \frac{1}{40} \cos 2\eta_1 \\
& + \operatorname{Re} \left\{ -e^{-2i\eta_1} F(2\eta_1, 0) + F(2\eta_1, -2) \right\} - 2 C_2^{(6)}(2, \eta_1) + \frac{\pi^2}{12} - \frac{29}{35} \\
& + \frac{9}{2} \times \\
& + \operatorname{Re} \left\{ e^{-2i\eta_1} F(2\eta_1, 2) - 2 F(2\eta_1, 0) + e^{2i\eta_1} F(2\eta_1, -2) \right\} - \\
& \quad - \cos 2\eta_1 \left\{ \frac{\cos 8\eta_1}{64} + \frac{\cos 10\eta_1}{100} - \frac{1}{1600} \right\} + \sin 2\eta_1 \left\{ S_2^{(10)}(2, \eta_1) + S_2^{(6)}(2, \eta_1) \right\} \\
& + \operatorname{Re} \left\{ -\frac{1}{8} e^{-4i\eta_1} F(2\eta_1, +2) + \frac{1}{3} e^{-2i\eta_1} F(2\eta_1, 0) - \frac{1}{4} F(2\eta_1, -2) + \right. \\
& \quad \left. + \frac{1}{24} e^{4i\eta_1} F(2\eta_1, -6) \right\} + \frac{11}{2880} \cos 4\eta_1 \\
& + \operatorname{Re} \left\{ -\frac{1}{12} e^{-2i\eta_1} F(2\eta_1, 6) - \frac{1}{4} e^{2i\eta_1} F(2\eta_1, 2) + \frac{1}{3} e^{4i\eta_1} F(2\eta_1, 0) \right\} - \\
& \quad - \cos 2\eta_1 \left\{ C_2^{(10)}(2, \eta_1) - \frac{1}{24} \pi^2 + \frac{41543}{100800} \right\} + \sin 2\eta_1 S_2^{(10)}(\eta_1) \\
& + \operatorname{Re} \left\{ F(2\eta_1, 2) - e^{+2i\eta_1} F(2\eta_1, 0) \right\} + 4 C_3^{(10)}(2, \eta_1) + 2 C_2^{(10)}(2, \eta_1) - \\
& \quad - \frac{1}{2} \mathcal{J}(3) - \frac{\pi^2}{12} + \frac{615443}{432000} \\
& + \operatorname{Re} \left\{ -e^{-2i\eta_1} F(2\eta_1, 0) + F(2\eta_1, -2) \right\} + 4 C_3^{(6)}(2, \eta_1) - 2 C_2^{(6)}(2, \eta_1) - \\
& \quad - \frac{1}{2} \mathcal{J}(3) + \frac{\pi^2}{12} - \frac{97}{432}
\end{aligned}$$

$$\begin{aligned}
& + \operatorname{Re} \left\{ \frac{1}{3} e^{-4i\eta_1} F(2\eta_1, 0) - \frac{1}{4} e^{-2i\eta_1} F(2\eta_1, -2) - \frac{1}{12} e^{2i\eta_1} F(2\eta_1, -6) \right\} + \\
& \quad + \cos 2\eta_1 \left\{ C_2^{(6)}(2, \eta_1) - \frac{\pi^2}{24} + \frac{5}{12} \right\} + \sin 2\eta_1 S_2^{(6)}(2, \eta_1) \\
& + \operatorname{Re} \left\{ \frac{1}{24} e^{-4i\eta_1} F(2\eta_1, 6) - \frac{1}{4} F(2\eta_1, 2) + \frac{1}{3} e^{2i\eta_1} F(2\eta_1, 0) - \frac{1}{8} e^{4i\eta_1} F(2\eta_1, -2) \right\} + \\
& \quad + \frac{5}{4032} \cos 4\eta_1 \}
\end{aligned}$$

$$3. \quad \underline{8 \sum_{n=5}^{\infty} - \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)}(0)} T_n(\beta, n, \eta_1)^* S_n(\beta, \eta_1) .}$$

$$\begin{aligned}
& \operatorname{Re} \left\{ \frac{1}{2} F(2\eta_1, 4) e^{-i\eta_1} - F(2\eta_1, 2) e^{i\eta_1} + \frac{1}{2} F(2\eta_1, 0) e^{3i\eta_1} \right\} - \frac{1}{120} \cos \eta_1 + \\
& + \operatorname{Re} \left\{ -\frac{1}{6} F(2\eta_1, 4) e^{-3i\eta_1} + \frac{1}{2} F(2\eta_1, 0) e^{i\eta_1} - \frac{1}{3} F(2\eta_1, -2) e^{3i\eta_1} \right\} + \\
& \quad + \frac{1}{90} \cos 3\eta_1 + \\
& + \operatorname{Re} \left\{ -\frac{1}{3} F(2\eta_1, 2) e^{-3i\eta_1} + \frac{1}{2} F(2\eta_1, 0) e^{-i\eta_1} - \frac{1}{6} F(2\eta_1, -4) e^{3i\eta_1} \right\} + \\
& \quad + \frac{11}{720} \cos 3\eta_1 + \\
& + \operatorname{Re} \left\{ \frac{1}{2} F(2\eta_1, 0) e^{-3i\eta_1} - F(2\eta_1, -2) e^{-i\eta_1} + \frac{1}{2} F(2\eta_1, -4) e^{i\eta_1} - \right. \\
& \quad \left. - \frac{1}{48} \cos \eta_1 + \right. \\
& + i \times \frac{\beta^2 \Omega}{2} \\
& \left[\operatorname{Re} \left\{ \frac{3}{4} F(2\eta_1, 4) - F(2\eta_1, 2) e^{2i\eta_1} + \frac{1}{4} F(2\eta_1, 0) e^{4i\eta_1} \right\} + C_2^{(12)}(2, \eta_1) - \right. \\
& \quad \left. - \frac{\pi^2}{24} + \frac{5909}{14400} + \right. \\
& + \operatorname{Re} \left\{ \frac{1}{3} F(2\eta_1, 4) e^{-2i\eta_1} - F(2\eta_1, 2) + F(2\eta_1, 0) e^{2i\eta_1} - \frac{1}{3} F(2\eta_1, -2) e^{4i\eta_1} \right\} + \\
& \quad + \frac{1}{360} \cos 2\eta_1 + \\
& + \operatorname{Re} \left\{ -\frac{1}{24} F(2\eta_1, 4) e^{-4i\eta_1} + \frac{1}{4} F(2\eta_1, 0) - \frac{1}{3} F(2\eta_1, -2) e^{2i\eta_1} + \frac{1}{8} F(2\eta_1, -4) e^{4i\eta_1} \right\} - \\
& \quad - \frac{7}{2880} \cos 4\eta_1 +
\end{aligned}$$

$$\begin{aligned}
& + \operatorname{Re} \left\{ \frac{1}{8} F(2\eta_1, 4) e^{-4i\eta_1} - \frac{1}{3} F(2\eta_1, 2) e^{-2i\eta_1} + \frac{1}{4} F(2\eta_1, 0) - \frac{1}{24} F(2\eta_1, -4) e^{4i\eta_1} \right\} + \\
& \quad + \frac{1}{576} \cos 4\eta_1 + \\
& + \operatorname{Re} \left\{ -\frac{1}{3} F(2\eta_1, 2) e^{-4i\eta_1} + F(2\eta_1, 0) e^{-2i\eta_1} - F(2\eta_1, -2) + \frac{1}{3} F(2\eta_1, -4) e^{2i\eta_1} \right\} - \\
& \quad - \frac{1}{180} \cos 2\eta_1 + \\
& + \operatorname{Re} \left\{ \frac{1}{4} F(2\eta_1, 0) e^{-4i\eta_1} - F(2\eta_1, -2) e^{-2i\eta_1} + \frac{3}{4} F(2\eta_1, -4) \right\} - \\
& \quad - C_2^{(4)}(2, \eta_1) + \frac{\pi^2}{24} - \frac{13}{32} + \\
& + \operatorname{Re} \left\{ \frac{1}{6} F(2\eta_1, 6) e^{-2i\eta_1} - \frac{1}{2} F(2\eta_1, 4) + \frac{1}{2} F(2\eta_1, 2) e^{2i\eta_1} - \frac{1}{6} F(2\eta_1, 0) e^{4i\eta_1} \right\} + \\
& \quad + \frac{1}{1260} \cos 2\eta_1 + \\
& + \operatorname{Re} \left\{ -\frac{1}{2} F(2\eta_1, 4) e^{-2i\eta_1} + \frac{1}{2} F(2\eta_1, 0) e^{2i\eta_1} \right\} - 2 C_2^{(10)}(2, \eta_1) + \frac{\pi^2}{12} - \frac{5929}{7200} + \\
& + \operatorname{Re} \left\{ -\frac{1}{6} F(2\eta_1, 2) e^{-2i\eta_1} + \frac{1}{2} F(2\eta_1, 0) - \frac{1}{2} F(2\eta_1, -2) e^{2i\eta_1} + \right. \\
& \quad \left. + \frac{1}{6} F(2\eta_1, -4) e^{4i\eta_1} \right\} - \frac{1}{360} \cos 2\eta_1 + \\
& + \operatorname{Re} \left\{ \frac{1}{12} F(2\eta_1, 2) e^{-4i\eta_1} - \frac{1}{6} F(2\eta_1, 0) e^{-2i\eta_1} + \frac{1}{6} F(2\eta_1, -4) e^{2i\eta_1} - \right. \\
& \quad \left. - \frac{1}{12} F(2\eta_1, -6) e^{4i\eta_1} \right\} + \frac{7}{1440} \cos 4\eta_1 + \\
& + \operatorname{Re} \left\{ -\frac{1}{12} F(2\eta_1, 6) e^{-4i\eta_1} + \frac{1}{6} F(2\eta_1, 4) e^{-2i\eta_1} - \frac{1}{6} F(2\eta_1, 0) e^{2i\eta_1} + \right. \\
& \quad \left. + \frac{1}{12} F(2\eta_1, -2) e^{4i\eta_1} \right\} - \frac{11}{10080} \cos 4\eta_1 + \\
& + \operatorname{Re} \left\{ \frac{1}{6} F(2\eta_1, 4) e^{-4i\eta_1} - \frac{1}{2} F(2\eta_1, 2) e^{-2i\eta_1} + \frac{1}{2} F(2\eta_1, 0) - \frac{1}{6} F(2\eta_1, -2) e^{2i\eta_1} \right\} + \\
& \quad + \frac{1}{720} \cos 2\eta_1 + \\
& + \operatorname{Re} \left\{ \frac{1}{2} F(2\eta_1, 0) e^{-2i\eta_1} - \frac{1}{2} F(2\eta_1, -4) e^{2i\eta_1} \right\} + 2 C_2^{(6)}(2, \eta_1) - \frac{\pi^2}{12} + \frac{119}{144} + \\
& + \operatorname{Re} \left\{ -\frac{1}{6} F(2\eta_1, 0) e^{-4i\eta_1} + \frac{1}{2} F(2\eta_1, -2) e^{-2i\eta_1} + \frac{1}{2} F(2\eta_1, -4) + \right. \\
& \quad \left. + \frac{1}{6} F(2\eta_1, -6) e^{2i\eta_1} \right\} - \frac{1}{144} \cos 2\eta_1 \Big] + \\
& + \frac{9}{4} \times \\
& \left[\operatorname{Re} \left\{ -\frac{4}{9} F(2\eta_1, 2) e^{-3i\eta_1} + \frac{1}{2} F(2\eta_1, 0) e^{-i\eta_1} - \frac{1}{18} F(2\eta_1, -4) e^{3i\eta_1} \right\} - \right. \\
& \quad \left. - \cos 3\eta_1 \left[\frac{2}{3} C_2^{(10)}(2, \eta_1) - \frac{\pi^2}{36} + \frac{5879}{21600} \right] - \frac{2}{3} S_2^{(10)}(2\eta_1) \sin 3\eta_1 + \right.
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Re} \left\{ \frac{3}{2} F(2\eta_1, 2) e^{-i\eta_1} - 2 F(2\eta_1, 0) e^{i\eta_1} + \frac{1}{2} F(2\eta_1, -2) e^{3i\eta_1} \right\} - \\
& \quad - \cos \eta_1 \left(-2 C_2^{(10)}(2, \eta_1) + \frac{\pi^2}{12} - \frac{5899}{7200} \right) + 2 S_2^{(10)}(2, \eta_1) \sin \eta_1 + \\
& \operatorname{Re} \left\{ \frac{1}{36} F(2\eta_1, 8) e^{-3i\eta_1} + \frac{2}{9} F(2\eta_1, 2) e^{3i\eta_1} - \frac{1}{4} F(2\eta_1, 0) e^{5i\eta_1} \right\} - \\
& \quad - \cos 3\eta_1 \left(-\frac{2}{3} C_2^{(10)}(2, \eta_1) + \frac{\pi^2}{36} - \frac{9239}{33600} \right) - \frac{2}{3} S_2^{(10)}(2, \eta_1) \sin 3\eta_1 + \\
& \operatorname{Re} \left\{ \frac{1}{10} F(2\eta_1, 2) e^{-5i\eta_1} - \frac{1}{4} F(2\eta_1, 0) e^{-3i\eta_1} + \frac{1}{6} F(2\eta_1, -2) e^{-i\eta_1} - \right. \\
& \quad \left. - \frac{1}{60} F(2\eta_1, 8) e^{5i\eta_1} \right\} + \frac{47}{7200} \cos 5\eta_1 + \\
& \operatorname{Re} \left\{ \frac{1}{2} F(2\eta_1, 2) e^{-3i\eta_1} - 2 F(2\eta_1, 0) e^{-i\eta_1} + \frac{3}{2} F(2\eta_1, -2) e^{i\eta_1} \right\} - \\
& \quad - \cos \eta_1 \left(2 C_2^{(6)}(2, \eta_1) - \frac{\pi^2}{12} + \frac{589}{720} \right) + 2 S_2^{(6)}(2, \eta_1) \sin \eta_1 + \\
& \operatorname{Re} \left\{ -\frac{1}{60} F(2\eta_1, 8) e^{-5i\eta_1} + \frac{1}{6} F(2\eta_1, 2) e^{i\eta_1} - \frac{1}{4} F(2\eta_1, 0) e^{3i\eta_1} + \right. \\
& \quad \left. + \frac{1}{10} F(2\eta_1, -2) e^{5i\eta_1} \right\} - \cos 5\eta_1 \cdot \frac{113}{100800} + \\
& \operatorname{Re} \left\{ -\frac{1}{4} F(2\eta_1, 0) e^{-5i\eta_1} + \frac{2}{9} F(2\eta_1, -2) e^{-3i\eta_1} + \frac{1}{36} F(2\eta_1, -8) e^{3i\eta_1} \right\} - \\
& \quad - \cos 3\eta_1 \left(\frac{2}{3} C_2^{(6)}(2, \eta_1) - \frac{\pi^2}{36} + \frac{245}{864} \right) - \frac{2}{3} S_2^{(6)}(2, \eta_1) \sin 3\eta_1 + \\
& \operatorname{Re} \left\{ -\frac{1}{18} F(2\eta_1, 4) e^{-3i\eta_1} + \frac{1}{2} F(2\eta_1, 0) e^{i\eta_1} - \frac{4}{9} F(2\eta_1, -2) e^{3i\eta_1} \right\} - \\
& \quad - \cos 3\eta_1 \left(-\frac{2}{3} C_2^{(6)}(2, \eta_1) + \frac{\pi^2}{36} - \frac{49}{180} \right) - \frac{2}{3} S_2^{(6)}(2, \eta_1) \sin 3\eta_1 + \\
& \operatorname{Re} \left\{ \frac{1}{30} F(2\eta_1, 4) e^{-5i\eta_1} - \frac{1}{6} F(2\eta_1, 0) e^{-i\eta_1} + \frac{1}{6} F(2\eta_1, -2) e^{i\eta_1} - \right. \\
& \quad \left. - \frac{1}{30} F(2\eta_1, -6) e^{5i\eta_1} \right\} + \frac{1}{300} \cos 5\eta_1 + \\
& \operatorname{Re} \left\{ \frac{1}{6} F(2\eta_1, 6) e^{-i\eta_1} - \frac{1}{2} F(2\eta_1, 4) e^{i\eta_1} + \frac{1}{2} F(2\eta_1, 2) e^{3i\eta_1} - \frac{1}{6} F(2\eta_1, 0) e^{5i\eta_1} \right\} + \\
& \quad + \frac{1}{1260} \cos \eta_1 + \\
& \operatorname{Re} \left\{ -\frac{1}{2} F(2\eta_1, 4) e^{-i\eta_1} + \frac{1}{2} F(2\eta_1, 0) e^{3i\eta_1} \right\} - \cos \eta_1 \left(+2 C_2^{(10)}(2, \eta_1) - \right. \\
& \quad \left. - \frac{\pi^2}{12} + \frac{5929}{7200} \right) + 2 S_2^{(10)}(2, \eta_1) \sin \eta_1 + \\
& \operatorname{Re} \left\{ \frac{1}{2} F(2\eta_1, 0) e^{-3i\eta_1} - \frac{1}{2} F(2\eta_1, -4) e^{i\eta_1} \right\} - \cos \eta_1 \left(-2 C_2^{(6)}(2, \eta_1) + \frac{\pi^2}{12} - \frac{119}{144} \right) + \\
& \quad + 2 S_2^{(6)}(2, \eta_1) \sin \eta_1 +
\end{aligned}$$

$$\begin{aligned} \text{Re} \left\{ -\frac{1}{6} F(2\eta_1, 0) e^{-5i\eta_1} + \frac{1}{2} F(2\eta_1, -2) e^{-3i\eta_1} - \frac{1}{2} F(2\eta_1, -4) e^{-i\eta_1} + \right. \\ \left. + \frac{1}{6} F(2\eta_1, -6) e^{i\eta_1} \right\} - \frac{1}{144} \cos \eta_1 + \\ \text{Re} \left\{ -\frac{1}{30} F(2\eta_1, 6) e^{-5i\eta_1} + \frac{1}{6} F(2\eta_1, 2) e^{-i\eta_1} - \frac{1}{6} F(2\eta_1, 0) e^{i\eta_1} + \right. \\ \left. + \frac{1}{30} F(2\eta_1, -4) e^{5i\eta_1} \right\} - \frac{13}{8400} \cos 5\eta_1 \end{aligned}$$

The complex conjugated of this result gives the result of

$$8. \sum_n^{\infty} \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} S_n(\beta, \Omega, \eta_1) T_n^*(\beta, \Omega, \eta_1) .$$

$$10: -16x \sum_n^{\infty} \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)'}(0)} T_n(\beta, \Omega, \eta_1) T_n^*(\beta, \Omega, \eta_1) .$$

$$\begin{aligned} & \text{Re} \left\{ \frac{1}{4} F(2\eta_1, 4) - \frac{1}{4} e^{4i\eta_1} F(2\eta_1, 0) \right\} + C_2^{(12)}(2, \eta_1) - \frac{\pi^2}{24} + \frac{6029}{14400} + \\ & + \text{Re} \left\{ \frac{1}{4} e^{-4i\eta_1} F(2\eta_1, 4) - \frac{1}{2} F(2\eta_1, 0) + \frac{1}{4} e^{4i\eta_1} F(2\eta_1, -4) \right\} - \frac{13}{480} \cos 4\eta_1 + \\ & + \text{Re} \left\{ -\frac{1}{4} e^{-4i\eta_1} F(2\eta_1, 0) + \frac{1}{4} F(2\eta_1, -4) \right\} - C_2^{(4)}(2, \eta_1) + \frac{\pi^2}{24} - \frac{37}{96} \\ & + 9/4 x \\ & \left[+ \text{Re} \left\{ \frac{2}{3} e^{-2i\eta_1} F(2\eta_1, 4) - 2 F(2\eta_1, 2) + 2 e^{2i\eta_1} F(2\eta_1, 0) - \frac{2}{3} e^{4i\eta_1} F(2\eta_1, -2) \right\} + \right. \\ & \qquad \qquad \qquad \left. + \frac{1}{180} \cos 2\eta_1 + \right. \\ & + \text{Re} \left\{ -\frac{1}{18} e^{-6i\eta_1} F(2\eta_1, 4) + \frac{1}{4} e^{-2i\eta_1} F(2\eta_1, 0) - \frac{2}{9} F(2\eta_1, -2) + \right. \\ & \qquad \qquad \qquad \left. + \frac{1}{36} e^{6i\eta_1} F(2\eta_1, -8) \right\} - \frac{49}{4320} \cos 6\eta_1 + \\ & + \text{Re} \left\{ -\frac{1}{12} e^{-2i\eta_1} F(2\eta_1, 8) + \frac{1}{2} e^{2i\eta_1} F(2\eta_1, 4) - \frac{2}{3} e^{4i\eta_1} F(2\eta_1, 2) + \right. \\ & \qquad \qquad \qquad \left. + \frac{1}{4} e^{6i\eta_1} F(2\eta_1, 0) \right\} - \frac{29}{20160} \cos 2\eta_1 + \\ & + \text{Re} \left\{ -\frac{2}{3} e^{-4i\eta_1} F(2\eta_1, 2) + 2 e^{-2i\eta_1} F(2\eta_1, 0) - 2 F(2\eta_1, -2) + \frac{2}{3} e^{2i\eta_1} F(2\eta_1, -4) \right\} \\ & \qquad \qquad \qquad - \frac{1}{90} \cos 2\eta_1 + \end{aligned}$$

$$\begin{aligned}
& + \operatorname{Re} \left\{ + \frac{1}{4} e^{-6i\eta_1} F(2\eta_1, 0) - \frac{2}{3} e^{-4i\eta_1} F(2\eta_1, -2) + \frac{1}{2} e^{-2i\eta_1} F(2\eta_1, -4) - \right. \\
& \quad \left. - \frac{1}{12} e^{2i\eta_1} F(2\eta_1, -8) \right\} + \frac{7}{288} \cos 2\eta_1 + \\
& + \operatorname{Re} \left\{ \frac{1}{36} e^{-6i\eta_1} F(2\eta_1, 8) - \frac{2}{9} F(2\eta_1, 2) + \frac{1}{4} e^{2i\eta_1} F(2\eta_1, 0) - \frac{1}{18} e^{6i\eta_1} F(2\eta_1, -4) \right\} + \\
& \quad + \frac{169}{60480} \cos 6\eta_1 \Big]
\end{aligned}$$

$$11: 2 \sum_{n=0}^{\infty} \Lambda_n(\beta, \Omega) S_n^*(\beta, \Omega, \eta_1) .$$

$$A^* \left[\Theta_0 x \right.$$

$$\operatorname{Im} \left\{ G(\eta_1, 0) \cdot (e^{i\eta_1} + e^{-i\eta_1}) - G(\eta_1, 1) - G(\eta_1, -1) \right\} .$$

$$- \frac{1}{2} \Theta_1 x$$

$$\operatorname{Im} \left\{ G(\eta_1, -1) \cdot (1 - e^{2i\eta_1}) + G(\eta_1, 1) \cdot (1 - e^{-2i\eta_1}) \right\} + 2 \left\{ \bar{S}_2^{(8)}(\eta_1) - \bar{S}_2^{(6)}(\eta_1) \right\} .$$

$$- \frac{1}{3} \Theta_2 x$$

$$\operatorname{Im} \left\{ -2 G(\eta_1, 1) - 2 G(\eta_1, -1) + G(\eta_1, 2) \cdot (3 e^{-i\eta_1} - e^{-3i\eta_1}) + G(\eta_1, -2) \cdot \right.$$

$$\left. (3 e^{i\eta_1} - e^{3i\eta_1}) \right\} .$$

$$+ \frac{1}{4} \Theta_3 x$$

$$\operatorname{Im} \left\{ G(\eta_1, 1) + G(\eta_1, -1) + G(\eta_1, 3) \cdot (-2 e^{-2i\eta_1} + e^{-4i\eta_1}) + G(\eta_1, -3) \cdot \right.$$

$$\left. (-2 e^{2i\eta_1} + e^{4i\eta_1}) \right\} .$$

$$+ \frac{1}{15} \Theta_4 x$$

$$\operatorname{Im} \left\{ 2 G(\eta_1, 1) + 2 G(\eta_1, -1) + G(\eta_1, 4) \cdot (-5 e^{-3i\eta_1} + 3 e^{-5i\eta_1}) + \right.$$

$$\left. + G(\eta_1, -4) \cdot (-5 e^{3i\eta_1} + 3 e^{5i\eta_1}) \right\} .$$

$$- \frac{1}{12} \Theta_5 x$$

$$\operatorname{Im} \left\{ -G(\eta_1, 1) - G(\eta_1, -1) + G(\eta_1, 5) \cdot (3 e^{-4i\eta_1} - 2 e^{-6i\eta_1}) + G(\eta_1, -5) \cdot \right.$$

$$\left. (3 e^{4i\eta_1} - 2 e^{6i\eta_1}) \right\}$$

$$- \frac{1}{35} \Theta_6 x$$

$$\operatorname{Im} \left\{ -2 G(\eta_1, 1) - 2 G(\eta_1, -1) + G(\eta_1, 6) \cdot (7 e^{-5i\eta_1} - 5 e^{-7i\eta_1}) + \right.$$

$$\left. + G(\eta_1, -6) \cdot (7 e^{5i\eta_1} - 5 e^{7i\eta_1}) \right\}$$

$$\begin{aligned}
& + \frac{1}{24} \Theta_7 x \\
& \quad \text{Im} \left\{ G(\eta_1, 1) + G(\eta_1, -1) + G(\eta_1, 7) \cdot (-4 e^{-6i\eta_1} + 3 e^{-8i\eta_1}) + \right. \\
& \quad \quad \left. + G(\eta_1, -7) \cdot (-4 e^{6i\eta_1} + 3 e^{8i\eta_1}) \right\} \Bigg] \\
& + \frac{i\beta^2 \Omega}{2} A^* \left[\frac{1}{2} \Theta_0 x \right. \\
& \quad \text{Im} \left\{ G(\eta_1, -2) + G(\eta_1, 2) + G(\eta_1, -1) \cdot (-2 e^{-i\eta_1} - 2 e^{i\eta_1}) + \right. \\
& \quad \quad \left. + G(\eta_1, 1) \cdot (-2 e^{i\eta_1} - 2 e^{-i\eta_1}) + G(\eta_1, 0) \cdot (e^{2i\eta_1} + 4 + e^{-2i\eta_1}) \right\} \\
& + \frac{1}{6} \Theta_1 x \\
& \quad \text{Im} \left\{ 8 G(\eta_1, -2) + G(\eta_1, -1) \cdot (-9 e^{-i\eta_1} + e^{3i\eta_1}) \right. \\
& \quad \quad \left. + 8 G(\eta_1, 2) + G(\eta_1, 1) \cdot (-9 e^{i\eta_1} + e^{-3i\eta_1}) \right\} + 12 \cos \eta_1 \cdot \left(\bar{S}_2^{(6)}(\eta_1) - \bar{S}_2^{(8)}(\eta_1) \right). \\
& - \frac{1}{12} \Theta_2 x \\
& \quad \text{Im} \left\{ G(\eta_1, -2) \cdot (9 + 8e^{2i\eta_1} - e^{4i\eta_1}) + G(\eta_1, -1) \cdot (-8 e^{-i\eta_1} - 8 e^{i\eta_1}) + \right. \\
& \quad \quad \left. + G(\eta_1, 2) \cdot (9 + 8 e^{-2i\eta_1} - e^{-4i\eta_1}) + G(\eta_1, 1) \cdot (-8 e^{i\eta_1} - 8 e^{-i\eta_1}) \right\} + \\
& \quad \quad \quad + 12 \left(\bar{S}_2^{(9)}(\eta_1) - \bar{S}_{(2)}^{(5)}(\eta_1) \right). \\
& - \frac{1}{20} \Theta_3 x \\
& \quad \text{Im} \left\{ G(\eta_1, -3) \cdot (-10 e^{i\eta_1} + 5 e^{3i\eta_1} - e^{5i\eta_1}) + 16 G(\eta_1, -2) + \right. \\
& \quad \quad \quad + G(\eta_1, -1) \cdot (-5 e^{i\eta_1} - 5 e^{-i\eta_1}) + \\
& \quad \quad \quad G(\eta_1, 3) \cdot (-10 e^{-i\eta_1} + 5 e^{-3i\eta_1} - e^{-5i\eta_1}) + 16 G(\eta_1, 2) + \\
& \quad \quad \quad \left. + G(\eta_1, 1) \cdot (-5 e^{-i\eta_1} - 5 e^{i\eta_1}) \right\}. \\
& + \frac{1}{30} \Theta_4 x \\
& \quad \text{Im} \left\{ G(\eta_1, -4) \cdot (+5e^{2i\eta_1} - 4e^{4i\eta_1} + e^{6i\eta_1}) - 10 G(\eta_1, -2) + \right. \\
& \quad \quad \quad + G(\eta_1, -1) \cdot (4 e^{-i\eta_1} + 4 e^{i\eta_1}) + \\
& \quad \quad \quad G(\eta_1, 4) \cdot (+5e^{-2i\eta_1} - 4e^{-4i\eta_1} + e^{-6i\eta_1}) - 10 G(\eta_1, 2) + \\
& \quad \quad \quad \left. + G(\eta_1, 1) \cdot (4 e^{-i\eta_1} + 4 e^{i\eta_1}) \right\}.
\end{aligned}$$

$$+ \frac{1}{84} \Theta_5 x$$

$$\begin{aligned} \operatorname{Im} \{ & G(\eta_1, -5) \cdot (7 e^{3i\eta_1} - 7 e^{5i\eta_1} + 2 e^{7i\eta_1}) - 16 G(\eta_1, -2) + \\ & + G(\eta_1, -1) \cdot (7 e^{i\eta_1} + 7 e^{-i\eta_1}) + \\ & + G(\eta_1, 5) \cdot (7 e^{-3i\eta_1} - 7 e^{-5i\eta_1} + 2 e^{-7i\eta_1}) - 16 G(\eta_1, 2) + \\ & + G(\eta_1, 1) \cdot (7 e^{i\eta_1} + 7 e^{-i\eta_1}) \} \end{aligned}$$

$$+ c/4 A^* \left[\frac{1}{6} \Theta_6 x \right.$$

$$\begin{aligned} \operatorname{Im} \{ & - G(\eta_1, -3) + G(\eta_1, -2) \cdot (-6 e^{i\eta_1} + 2 e^{3i\eta_1}) + G(\eta_1, -1) \cdot (3 e^{-2i\eta_1} + 1 - 3 e^{2i\eta_1}) + \\ & - G(\eta_1, 3) + G(\eta_1, 2) \cdot (-6 e^{-i\eta_1} + 2 e^{-3i\eta_1}) + G(\eta_1, 1) \cdot (3 e^{2i\eta_1} + 1 - 3 e^{-2i\eta_1}) + \\ & G(\eta_1, 0) \cdot (6 e^{-i\eta_1} - 2 e^{-3i\eta_1} - 2 e^{3i\eta_1} + 6 e^{i\eta_1}) \} + 12 \left(\bar{S}_2^{(8)}(\eta_1) - \bar{S}_2^{(6)}(\eta_1) \right). \end{aligned}$$

$$- \frac{1}{8} \Theta_1 x$$

$$\begin{aligned} \operatorname{Im} \{ & G(\eta_1, -1) \cdot (-3 - 4 e^{-2i\eta_1} + 2 e^{2i\eta_1} + e^{4i\eta_1}) + G(\eta_1, -3) \cdot (3 + 2 e^{2i\eta_1} - e^{4i\eta_1}) \\ & + G(\eta_1, 1) \cdot (-3 - 4 e^{2i\eta_1} + 2 e^{-2i\eta_1} + e^{-4i\eta_1}) + G(\eta_1, 3) \cdot (3 + 2 e^{-2i\eta_1} - e^{-4i\eta_1}) \} + \\ & + 8(1 + \cos 2\eta_1) \cdot (\bar{S}_2^{(6)}(\eta_1) - \bar{S}_2^{(8)}(\eta_1)). \end{aligned}$$

$$- \frac{1}{45} \Theta_2 x$$

$$\begin{aligned} \operatorname{Im} \{ & G(\eta_1, -4) \cdot (5 e^{3i\eta_1} - 3 e^{5i\eta_1}) + 27 G(\eta_1, -3) + \\ & + G(\eta_1, -2) \cdot (-45 e^{-i\eta_1} - 45 e^{i\eta_1} - 5 e^{3i\eta_1} + 3 e^{5i\eta_1}) + \\ & G(\eta_1, 4) \cdot (5 e^{-3i\eta_1} - 3 e^{-5i\eta_1}) + 27 G(\eta_1, 3) + \end{aligned}$$

$$\begin{aligned} & + G(\eta_1, +2) \cdot (-45 e^{i\eta_1} - 45 e^{-i\eta_1} - 5 e^{-3i\eta_1} + 3 e^{-5i\eta_1}) + \\ & + G(\eta_1, -1) \cdot (15 e^{-2i\eta_1} - 27 - 15 e^{2i\eta_1}) + (90 e^{i\eta_1} + 90 e^{-i\eta_1}) \cdot G(\eta_1, 0) + \\ & + G(\eta_1, 1) \cdot (-15 e^{-2i\eta_1} - 27 + 15 e^{2i\eta_1}) \} + 60 \left(\bar{S}_2^{(8)}(\eta_1) - \bar{S}_2^{(6)}(\eta_1) \right) \end{aligned}$$

$$+ \frac{1}{48} \Theta_3 x$$

$$\begin{aligned} \operatorname{Im} \{ & G(\eta_1, -5) \cdot (-3 e^{4i\eta_1} + 2 e^{6i\eta_1}) + G(\eta_1, -3) \cdot (8 + 12 e^{2i\eta_1} + 3 e^{4i\eta_1} - 2 e^{6i\eta_1}) + \\ & + G(\eta_1, -1) \cdot (-8 - 6 e^{-2i\eta_1} - 6 e^{2i\eta_1}) + \\ & + G(\eta_1, 5) \cdot (-3 e^{-4i\eta_1} + 2 e^{-6i\eta_1}) + G(\eta_1, +3) \cdot (8 + 12 e^{-2i\eta_1} + 3 e^{-4i\eta_1} - 2 e^{-6i\eta_1}) + \\ & + G(\eta_1, 1) \cdot (-8 - 6 e^{-2i\eta_1} - 6 e^{2i\eta_1}) \} + 24 \left(\bar{S}_2^{(4)}(\eta_1) - \bar{S}_2^{(10)}(\eta_1) \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{3150} \Theta_4 x, \\
& \operatorname{Im} \left\{ G(\eta_1, -6) \cdot (-126 e^{5i\eta_1} + 90 e^{7i\eta_1}) + \right. \\
& \quad + G(\eta_1, -4) \cdot (-1050 e^{i\eta_1} + 350 e^{3i\eta_1} + 126 e^{5i\eta_1} - 90 e^{7i\eta_1}) + 1350 G(\eta_1, -3) + \\
& \quad + G(\eta_1, 6) \cdot (-126 e^{-5i\eta_1} + 90 e^{-7i\eta_1}) + \\
& \quad + G(\eta_1, 4) \cdot (-1050 e^{-i\eta_1} + 350 e^{-3i\eta_1} + 126 e^{-5i\eta_1} - 90 e^{-7i\eta_1}) + 1350 G(\eta_1, 3) + \\
& \quad + G(\eta_1, -2) \cdot (1050 e^{i\eta_1} - 350 e^{3i\eta_1}) + G(\eta_1, -1) \cdot (210 e^{2i\eta_1} - 1350 - 210 e^{-2i\eta_1}) \\
& \quad + G(\eta_1, 2) \cdot (1050 e^{-i\eta_1} - 350 e^{-3i\eta_1}) + G(\eta_1, 1) \cdot (210 e^{-2i\eta_1} - 1350 - 210 e^{2i\eta_1}) \left. \right\} + \\
& \quad + 840 \left(\bar{S}_2^{(6)}(\eta_1) - \bar{S}_2^{(8)}(\eta_1) \right).
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{144} \Theta_5 x \\
& \operatorname{Im} \left\{ G(\eta_1, -7) \cdot (4 e^{6i\eta_1} - 3 e^{8i\eta_1}) + G(\eta_1, -5) \cdot (18 e^{2i\eta_1} - 9 e^{4i\eta_1} - 4 e^{6i\eta_1} + 3 e^{8i\eta_1}) + \right. \\
& \quad + G(\eta_1, 7) \cdot (4 e^{-6i\eta_1} - 3 e^{-8i\eta_1}) + G(\eta_1, 5) \cdot (18 e^{-2i\eta_1} - 9 e^{-4i\eta_1} - 4 e^{-6i\eta_1} + 3 e^{-8i\eta_1}) + \\
& \quad + G(\eta_1, -3) \cdot (-27 - 18 e^{2i\eta_1} + 9 e^{4i\eta_1}) + G(\eta_1, -1) \cdot (6 e^{-2i\eta_1} + 27 - 6 e^{2i\eta_1}) + \\
& \quad + G(\eta_1, 3) \cdot (-27 - 18 e^{-2i\eta_1} + 9 e^{-4i\eta_1}) + G(\eta_1, 1) \cdot (6 e^{2i\eta_1} + 27 - 6 e^{-2i\eta_1}) \left. \right\} + \\
& \quad + 24 \left(\bar{S}_2^{(8)}(\eta_1) - \bar{S}_2^{(6)}(\eta_1) \right) \Big].
\end{aligned}$$

$$12: \frac{4 \sum_n^{\infty} \Lambda_n(\beta, \Omega) T_n^*(\beta, \Omega, \eta_1)}{8}$$

$$\frac{i\beta^2\Omega}{12} A^* \left[\Theta_0 x \right]$$

$$\begin{aligned}
& \operatorname{Im} \left\{ G(\eta_1, 0) \cdot (-e^{3i\eta_1} + 3e^{i\eta_1} + 3e^{-i\eta_1} - e^{-3i\eta_1}) - 6 G(\eta_1, 1) - 6 G(\eta_1, -1) + \right. \\
& \quad + \left(G(\eta_1, 2) + G(\eta_1, -2) \right) \cdot (3e^{i\eta_1} + 3e^{-i\eta_1}) - 2G(\eta_1, 3) - 2 G(\eta_1, -3) \left. \right\}
\end{aligned}$$

$$+ \frac{1}{2} \Theta_1 x$$

$$\begin{aligned}
& \operatorname{Im} \left\{ G(\eta_1, 1) \cdot (-6e^{2i\eta_1} - 18 + 2e^{-2i\eta_1} - e^{-4i\eta_1}) + G(\eta_1, 2) \cdot (16e^{i\eta_1} + 16e^{-i\eta_1}) - \right. \\
& \quad - 9 G(\eta_1, 3) + \\
& \quad + G(\eta_1, -1) \cdot (-6e^{-2i\eta_1} - 18 + 2e^{2i\eta_1} - e^{4i\eta_1}) + G(\eta_1, -2) \cdot (16e^{i\eta_1} + 16e^{-i\eta_1}) - \\
& \quad - 9 G(\eta_1, -3) \left. \right\} + 12 \left(\bar{S}_2^{(6)}(\eta_1) - \bar{S}_2^{(8)}(\eta_1) \right).
\end{aligned}$$

$$- \frac{1}{10} \Theta_2 x$$

$$\begin{aligned} \operatorname{Im} \{ & -40 G(\eta_1, 1) + G(\eta_1, 2) \cdot (3e^{-5i\eta_1} - 5e^{-3i\eta_1} + 45e^{-i\eta_1} - 75e^{i\eta_1}) + 72 G(\eta_1, 3) + \\ & -40 G(\eta_1, -1) + G(\eta_1, -2) \cdot (3e^{5i\eta_1} - 5e^{3i\eta_1} + 45e^{i\eta_1} - 75e^{-i\eta_1}) + \\ & + 72 G(\eta_1, -3) \} + 120 \cos \eta_1 \left(\bar{S}_2^{(9)}(\eta_1) - \bar{S}_2^{(5)}(\eta_1) \right) \end{aligned}$$

$$- \frac{1}{10} \Theta_3 x$$

$$\begin{aligned} \operatorname{Im} \{ & -15 G(\eta_1, 1) + G(\eta_1, 2) \cdot (48e^{i\eta_1} + 48e^{-i\eta_1}) + G(\eta_1, 3) \cdot (-50 - 30e^{-2i\eta_1} - 3e^{-4i\eta_1} + 2e^{-6i\eta_1}) + \\ & -15 G(\eta_1, -1) + G(\eta_1, -2) \cdot (48e^{i\eta_1} + 48e^{-i\eta_1}) + \\ & + G(\eta_1, -3) \cdot (-50 - 30e^{2i\eta_1} - 3e^{4i\eta_1} + 2e^{6i\eta_1}) + 60 \left(\bar{S}_2^{(10)}(\eta_1) - \bar{S}_2^{(4)}(\eta_1) \right) \} \end{aligned}$$

$$+ \frac{1}{35} \Theta_4 x$$

$$\begin{aligned} \operatorname{Im} \{ & 28 G(\eta_1, 1) + G(\eta_1, 2) \cdot (-70e^{i\eta_1} - 70e^{-i\eta_1}) + \\ & + 180 G(\eta_1, 3) + (-105e^{-i\eta_1} + 35e^{-3i\eta_1} + 7e^{-5i\eta_1} - 5e^{-7i\eta_1}) G(\eta_1, 4) + \\ & + 28 G(\eta_1, -1) + G(\eta_1, -2) \cdot (-70e^{i\eta_1} - 70e^{-i\eta_1}) + \\ & + 180 G(\eta_1, -3) + (-105e^{i\eta_1} + 35e^{3i\eta_1} + 7e^{5i\eta_1} - 5e^{7i\eta_1}) G(\eta_1, -4) \} \end{aligned}$$

$$+ \frac{1}{28} \Theta_5 x$$

$$\operatorname{Im} \{ 14 G(\eta_1, -1) + G(\eta_1, -2) \cdot (-32e^{i\eta_1} - 32e^{-i\eta_1}) + 63 G(\eta_1, -3) +$$

$$\begin{aligned} & + G(\eta_1, -5) \cdot (-28e^{2i\eta_1} + 14e^{4i\eta_1} + 4e^{6i\eta_1} - 3e^{8i\eta_1}) \\ & + 14 G(\eta_1, 1) + G(\eta_1, 2) \cdot (-32e^{i\eta_1} - 32e^{-i\eta_1}) + 63 G(\eta_1, 3) + G(\eta_1, 5) \cdot (-28e^{-2i\eta_1} \\ & + 14e^{-4i\eta_1} + 4e^{-6i\eta_1} - 3e^{-8i\eta_1}) \} \end{aligned}$$

$$A^* \left[\frac{1}{2} \Theta_0 x \right]$$

$$\operatorname{Im} \{ G(\eta_1, 0) \cdot (e^{2i\eta_1} + e^{-2i\eta_1}) - G(\eta_1, 2) - G(\eta_1, -2) \}.$$

$$- \Theta_1 x$$

$$\begin{aligned} \operatorname{Im} \{ & G(\eta_1, 1) \cdot (-e^{i\eta_1} - \frac{1}{3} e^{-3i\eta_1}) + G(\eta_1, -1) \cdot (-e^{-i\eta_1} - \frac{1}{3} e^{3i\eta_1}) + \\ & + \frac{4}{3} G(\eta_1, 2) + \frac{4}{3} G(\eta_1, -2) \}. \end{aligned}$$

$$- \Theta_2 x$$

$$\operatorname{Im} \{ G(\eta_1, 2) \cdot \left(\frac{1}{4} - \frac{1}{4} e^{-4i\eta_1} \right) + G(\eta_1, -2) \cdot \left(\frac{1}{4} - \frac{1}{4} e^{4i\eta_1} \right) \} + \bar{S}_2^{(5)}(\eta_1) - \bar{S}_2^{(9)}(\eta_1).$$

$$+ \Theta_3 x$$

$$\begin{aligned} \operatorname{Im} \{ & \frac{4}{5} G(\eta_1, 2) + \frac{4}{5} G(\eta_1, -2) + G(\eta_1, 3) \cdot (-e^{-i\eta_1} + \frac{1}{5} e^{-5i\eta_1}) + \\ & + G(\eta_1, -3) \cdot (-e^{i\eta_1} + \frac{1}{5} e^{5i\eta_1}) \}. \end{aligned}$$

$$+ \Theta_4 x$$

$$\operatorname{Im} \left\{ \frac{1}{3} G(\eta_1, 2) + \frac{1}{3} G(\eta_1, -2) + G(\eta_1, 4) \cdot \left(-\frac{1}{2} e^{-2i\eta_1} + \frac{1}{6} e^{-6i\eta_1} \right) + \right.$$

$$\left. + G(\eta_1, -4) \cdot \left(-\frac{1}{2} e^{2i\eta_1} + \frac{1}{6} e^{6i\eta_1} \right) \right\}.$$

$$- \Theta_5 x$$

$$\operatorname{Im} \left\{ -\frac{4}{21} G(\eta_1, 2) - \frac{4}{21} G(\eta_1, -2) + G(\eta_1, 5) \cdot \left(\frac{1}{3} e^{-3i\eta_1} - \frac{1}{7} e^{-7i\eta_1} \right) + \right.$$

$$\left. + G(\eta_1, -5) \cdot \left(\frac{1}{3} e^{3i\eta_1} - \frac{1}{7} e^{7i\eta_1} \right) \right\}$$

$$- \Theta_6 x$$

$$\operatorname{Im} \left\{ -\frac{1}{8} G(\eta_1, 2) - \frac{1}{8} G(\eta_1, -2) + G(\eta_1, 6) \cdot \left(\frac{1}{4} e^{-4i\eta_1} - \frac{1}{8} e^{-8i\eta_1} \right) + \right.$$

$$\left. + G(\eta_1, -6) \cdot \left(\frac{1}{4} e^{4i\eta_1} - \frac{1}{8} e^{8i\eta_1} \right) \right\}$$

$$+ \Theta_7 x$$

$$\operatorname{Im} \left\{ \frac{4}{45} G(\eta_1, 2) + \frac{4}{45} G(\eta_1, -2) + G(\eta_1, 7) \cdot \left(-\frac{1}{5} e^{-5i\eta_1} + \frac{1}{9} e^{-9i\eta_1} \right) + \right.$$

$$\left. + G(\eta_1, -7) \cdot \left(-\frac{1}{5} e^{5i\eta_1} + \frac{1}{9} e^{9i\eta_1} \right) \right\}.$$

$$+ q/4 A^* \left[\Theta_0 x \right.$$

$$\operatorname{Im} \left\{ -G(\eta_1, 0) \cdot \left(\frac{1}{4} e^{4i\eta_1} + 2 + \frac{1}{4} e^{-4i\eta_1} \right) + G(\eta_1, 1) \cdot \left(-\frac{1}{3} e^{-3i\eta_1} + e^{-i\eta_1} - e^{i\eta_1} + \frac{1}{3} e^{3i\eta_1} \right) + \right.$$

$$\left. + G(\eta_1, -1) \cdot \left(+\frac{1}{3} e^{-3i\eta_1} - e^{-i\eta_1} + e^{i\eta_1} - \frac{1}{3} e^{3i\eta_1} \right) \right.$$

$$+ G(\eta_1, 2) \cdot \left(\frac{13}{12} + \frac{1}{4} e^{-4i\eta_1} \right) + G(\eta_1, -2) \cdot \left(\frac{13}{12} + \frac{1}{4} e^{4i\eta_1} \right) -$$

$$\left. - \frac{1}{12} G(\eta_1, 4) - \frac{1}{12} G(\eta_1, -4) \right\} + \bar{S}_2^{(9)}(\eta_1) - \bar{S}_2^{(5)}(\eta_1).$$

$$- \Theta_1 x$$

$$\operatorname{Im} \left\{ G(\eta_1, 1) \cdot \left(\frac{1}{10} e^{-5i\eta_1} + \frac{5}{18} e^{-3i\eta_1} - \frac{1}{2} e^{i\eta_1} - \frac{5}{18} e^{3i\eta_1} \right) + \right.$$

$$+ G(\eta_1, -1) \cdot \left(\frac{1}{10} e^{5i\eta_1} + \frac{5}{18} e^{3i\eta_1} - \frac{1}{2} e^{-i\eta_1} - \frac{5}{18} e^{-3i\eta_1} \right) +$$

$$- \frac{8}{45} G(\eta_1, 2) - \frac{8}{45} G(\eta_1, -2) + G(\eta_1, 3) \cdot \left(-\frac{1}{10} e^{-5i\eta_1} + \frac{1}{2} e^{-i\eta_1} \right) +$$

$$\left. + G(\eta_1, -3) \cdot \left(-\frac{1}{10} e^{5i\eta_1} + \frac{1}{2} e^{i\eta_1} \right) + \frac{8}{45} G(\eta_1, 4) + \frac{8}{45} G(\eta_1, -4) \right\}$$

$$+ 2 \cos \eta_1 \left(\bar{S}_2^{(6)}(\eta_1) - \bar{S}_2^{(8)}(\eta_1) \right) + \frac{2}{3} \cos 3\eta_1 \left(\bar{S}_2^{(6)}(\eta_1) - \bar{S}_2^{(8)}(\eta_1) \right).$$

$-\theta_2 \times$

$$\begin{aligned} & \operatorname{Im}\{G(\eta_1, 0) \cdot (e^{2i\eta_1} + e^{-2i\eta_1}) + G(\eta_1, 1) \cdot \left(\frac{2}{9}e^{3i\eta_1} - \frac{2}{3}e^{i\eta_1} + \frac{2}{3}e^{-i\eta_1} - \frac{2}{9}e^{-3i\eta_1}\right) + \\ & \quad + G(\eta_1, -1) \cdot \left(\frac{2}{9}e^{-3i\eta_1} - \frac{2}{3}e^{-i\eta_1} + \frac{2}{3}e^{i\eta_1} - \frac{2}{9}e^{3i\eta_1}\right) + \\ & \quad + G(\eta_1, 2) \cdot \left(-\frac{1}{2}e^{2i\eta_1} - \frac{2}{9} - \frac{2}{3}e^{-2i\eta_1} + \frac{1}{18}e^{-6i\eta_1}\right) + \\ & \quad + G(\eta_1, -2) \cdot \left(-\frac{1}{2}e^{-2i\eta_1} - \frac{2}{9} - \frac{2}{3}e^{2i\eta_1} + \frac{1}{18}e^{6i\eta_1}\right) + \\ & \quad + G(\eta_1, 4) \cdot \left(\frac{2}{9} + \frac{1}{6}e^{-2i\eta_1} - \frac{1}{18}e^{-6i\eta_1}\right) + G(\eta_1, -4) \cdot \left(\frac{2}{9} + \frac{1}{6}e^{2i\eta_1} - \frac{1}{18}e^{6i\eta_1}\right)\}. \end{aligned}$$

 $+\theta_3 \times$

$$\begin{aligned} & \operatorname{Im}\left\{\left(-\frac{1}{12}e^{-3i\eta_1} - \frac{1}{4}e^{-i\eta_1} - \frac{1}{4}e^{i\eta_1} - \frac{1}{12}e^{3i\eta_1}\right)(G(\eta_1, 1) + G(\eta_1, -1)) + \right. \\ & \quad + \frac{8}{21}(G(\eta_1, 2) + G(\eta_1, -2)) + \\ & \quad + G(\eta_1, 3) \cdot \left(\frac{1}{2}e^{i\eta_1} + \frac{1}{4}e^{-3i\eta_1} - \frac{1}{28}e^{-7i\eta_1}\right) + G(\eta_1, -3) \cdot \left(\frac{1}{2}e^{-i\eta_1} + \frac{1}{4}e^{3i\eta_1} - \frac{1}{28}e^{7i\eta_1}\right) - \\ & \quad - \frac{8}{21}(G(\eta_1, 4) + G(\eta_1, -4)) \\ & \quad \left. + G(\eta_1, 5) \cdot \left(-\frac{1}{12}e^{-3i\eta_1} + \frac{1}{28}e^{-7i\eta_1}\right) + G(\eta_1, -5) \cdot \left(-\frac{1}{12}e^{3i\eta_1} + \frac{1}{28}e^{7i\eta_1}\right)\right\} \end{aligned}$$

 $+\theta_4 \times$

$$\begin{aligned} & \operatorname{Im}\left\{\left(-\frac{2}{45}e^{3i\eta_1} + \frac{2}{15}e^{i\eta_1} - \frac{2}{15}e^{-i\eta_1} + \frac{2}{45}e^{-3i\eta_1}\right)(G(\eta_1, 1) - G(\eta_1, -1)) + \right. \\ & \quad + G(\eta_1, 2) \cdot \left(-\frac{5}{72} - \frac{1}{12}e^{-4i\eta_1}\right) + G(\eta_1, -2) \cdot \left(-\frac{5}{72} - \frac{1}{12}e^{4i\eta_1}\right) \\ & \quad + G(\eta_1, 4) \cdot \left(\frac{5}{72} + \frac{2}{15}e^{-4i\eta_1} - \frac{1}{40}e^{-8i\eta_1}\right) + G(\eta_1, -4) \cdot \left(\frac{5}{72} + \frac{2}{15}e^{4i\eta_1} - \frac{1}{40}e^{8i\eta_1}\right) + \\ & \quad + G(\eta_1, 6) \cdot \left(\frac{1}{40}e^{-8i\eta_1} - \frac{1}{20}e^{-4i\eta_1}\right) + G(\eta_1, -6) \cdot \left(\frac{1}{40}e^{8i\eta_1} - \frac{1}{20}e^{4i\eta_1}\right)\} + \\ & \quad + \frac{1}{3}\left(\bar{S} \binom{11}{2}(\eta_1) - \bar{S} \binom{9}{2}(\eta_1) + \bar{S} \binom{5}{2}(\eta_1) - \bar{S} \binom{3}{2}(\eta_1)\right). \end{aligned}$$

 $-\theta_5 \times$

$$\begin{aligned} & \operatorname{Im}\left\{\left(\frac{1}{3}e^{3i\eta_1} - e^{i\eta_1} + e^{-i\eta_1} - \frac{1}{3}e^{-3i\eta_1}\right)\frac{1}{12}(G(\eta_1, 1) - G(\eta_1, -1)) + \frac{8}{27}(G(\eta_1, 2) + G(\eta_1, -2)) + \right. \\ & \quad + G(\eta_1, 3) \cdot \left(-\frac{1}{4}e^{-i\eta_1} + \frac{1}{20}e^{-5i\eta_1}\right) + G(\eta_1, -3) \cdot \left(-\frac{1}{4}e^{i\eta_1} + \frac{1}{20}e^{5i\eta_1}\right) - \\ & \quad - \frac{8}{27}(G(\eta_1, 4) + G(\eta_1, -4)) + \\ & \quad + G(\eta_1, 5) \cdot \left(\frac{1}{4}e^{-i\eta_1} - \frac{1}{12}e^{-5i\eta_1} + \frac{1}{54}e^{-9i\eta_1}\right) + G(\eta_1, -5) \cdot \left(\frac{1}{4}e^{i\eta_1} - \frac{1}{12}e^{5i\eta_1} + \frac{1}{54}e^{9i\eta_1}\right) + \\ & \quad \left. + G(\eta_1, 7) \cdot \left(\frac{1}{30}e^{-5i\eta_1} - \frac{1}{54}e^{-9i\eta_1}\right) + G(\eta_1, -7) \cdot \left(\frac{1}{30}e^{5i\eta_1} - \frac{1}{54}e^{9i\eta_1}\right)\right\} \end{aligned}$$

Errata R 53, Int 7.

<u>Page</u>	<u>Line</u>	For:	read:
2	9	$se_m(\eta_1)$	$se_n(\eta_1)$
3	23	$\frac{\eta_1}{\sin 1}$	$\frac{\eta_1}{\sin \eta_1}$
	24	$\frac{\eta_1}{\sinh 1}$	$\frac{\eta_1}{\sin \eta_1}$
4	9	$\left\{ Jc\left(\frac{1}{c}, \frac{\Omega}{c}\right) - Ys\left(\frac{1}{c}, \frac{\Omega}{c}\right) \right\}$	$\frac{1}{c} \left\{ Jc\left(\frac{1}{c}, c\Omega\right) - Ys\left(\frac{1}{c}, c\Omega\right) \right\} .$
	10	$\left\{ Js\left(\frac{1}{c}, \frac{\Omega}{c}\right) + Yc\left(\frac{1}{c}, \frac{\Omega}{c}\right) \right\}$	$\frac{1}{c} \left\{ Js\left(\frac{1}{c}, c\Omega\right) + Yc\left(\frac{1}{c}, c\Omega\right) \right\} .$
5	21	$\sum_{j=0}^{\infty} x^{-j-1} dx$	$\sum_{j=0}^{\infty} \Lambda_j x^{-j-\ell} dx .$
6	4	$\frac{(i\alpha)^j}{j!}$	$\frac{(i\alpha)^j}{j!} .$
	9	$\frac{(i\alpha)^j}{j-1}$	$\frac{(i\alpha)^j}{j!} .$
7	7	$-\frac{\cos 4\tilde{z}}{16} \dots \frac{\cos 2k\tilde{z}}{(2k)^2}$	$-\frac{\cos 4\tilde{z}}{16} - \dots - \frac{\cos 2k\tilde{z}}{(2k)^2} .$
	10	$\frac{\cos 2n\tilde{z}}{(2n)^2}$	$\frac{\cos 2n\tilde{z}}{(2n)^3} .$
	13	$+\frac{\pi^2}{24}$	$+\frac{\pi^2\tilde{z}}{24} .$
	15	$+\frac{\cos \tilde{z}}{1} \frac{\cos 2\tilde{z}}{2^2} \dots +$	$+\frac{\cos \tilde{z}}{1} - \frac{\cos 2\tilde{z}}{2^2} + \dots +$
	17	$(2 \cos \tilde{z})$	$(2 \cos \tilde{z}/2) .$
	20	$(-1) \frac{\sin n\tilde{z}}{n^3}$	$(-1)^n \frac{\sin n\tilde{z}}{n^3} .$
8	8	"l being a positive"	"l being a non-negative"
	10		when $\alpha = -8 + 2\ell$;
	11	$\sum_{k=1}^{\ell}$	$\sum_{k=0}^{\ell}$
	22	$R_n(\beta^2, \Omega, \eta_1)$	$R_n(\beta^2 \Omega, \eta_1) .$
9	3	$(\beta^2, \Omega, \eta_1)$	$(\beta^2 \Omega, \eta_1)$ (twice).
	6	$\frac{R_{n+2}(\beta \Omega, \eta_1)}{(n+1)n}$	$\frac{R_{n+2}(\beta^2 \Omega, \eta_1)}{(n+1)n} .$

<u>Page</u>	<u>Line</u>	For:	Read:
9	11	$R_n(\beta, \Omega, \eta_1)$	$R_n(\beta^2 \Omega, \eta_1)$
	16	$(\beta^2, \Omega, \eta_1)$	$(\beta^2 \Omega, \eta_1)$ (twice)
	18	$+ \frac{\sin(n+2)\eta_1}{(n+1)(n+2)} +$	$+ \frac{\sin(n+2)\eta_1}{(n+1)(n+2)} \} +$
	21	$- R_{n+2}(\beta^2 \Omega, \eta_1) =$	$- R_{n+2}(\beta^2 \Omega, \eta_1) \} =$
10	9	$O(n^{-5})$	$O(n^{-5})$
	19	$R_n(\Omega)$	$R_n(\Omega)$
11	2	$-O(\tau^{\frac{1}{2}n} \cdot (\frac{n}{2})^{-\frac{n}{2}-1} \cdot \frac{1}{\Omega})$	$-O\left[\tau^{\frac{1}{2}n} \cdot (\frac{n}{2})^{-\frac{n}{2}-1} \cdot \frac{1}{\Omega}\right] \} \cdot$
12	2	$\frac{\theta_h}{n+h}$	$\frac{\theta_h}{N+h}$
	20	$\frac{e^{-m} \sin m \eta_1}{m} d \}$	$\frac{e^{-m} \sin m \eta_1}{m} \} d \}$
13	13	$-S_2^{(13)}(\eta_1)$	$-S_2^{(13)}(\eta_1)$
	15	$\{ 4e^{i\eta_1} - e^{i\eta_1} - e^{-3i\eta_1} \}$	$\{ 4e^{i\eta_1} - e^{-i\eta_1} - e^{-3i\eta_1} \}$
14	20	$q/4 \frac{\theta_7}{168} x$	$+ q/4 \frac{\theta_7}{168} x$
15	1	$q/4 \frac{\theta_8}{252} x$	$+ q/4 \frac{\theta_8}{252} x$
	15	$-4e^{-i\eta_1} F(2\eta_1, 0) -$	$-4e^{-i\eta_1} F(2\eta_1, 0) \} -$
16	1	$+ i q/4 \beta^2 \Omega A$	$+ i q/4 \beta^2 \Omega A x$
	17	$S_2^{(10)}(2\eta_1)$	$S_2^{(10)}(2, \eta_1) \cdot$
17	11	$- \frac{1}{4} S_2^{(6)}(2, \eta_1) \}$	$- \frac{1}{4} S_2^{(6)}(2, \eta_1) \} \}$
18	15	$+ \frac{1}{4} S_2^{(10)}(2, \eta_1) \}$	$+ \frac{1}{4} S_2^{(10)}(2, \eta_1) \} \}$
19	10	$+ \frac{2}{3} [\text{Re } e^{4i\eta_1} \dots$	$+ \frac{2}{3} [\text{Re } \{ e^{4i\eta_1} \dots$
21	16	$- 12 e^{4i\eta_1} F(2\eta_1, 8)$	$- 12 e^{4i\eta_1} F(2\eta_1, 0)$

Page	Line	For:	Read:
22	4	$2 e^{-4i\eta_1} F(2\eta_1, 0)$	$12 e^{-4i\eta_1} F(2\eta_1, 0)$
23	3	η should not be there.	
24	4	$-\frac{5921}{1200}]$	$-\frac{5921}{1200}]$
	10	$+ \operatorname{Re} \{ e^{-2i\eta} \dots$	$\{ + \operatorname{Re} \{ e^{-2i\eta} \dots$
	15	$S_2^{(10)}(\eta_1)$	$S_2^{(10)}(2, \eta_1)$
25	5	$8 \sum_{n=5}^{\infty} - \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)}(0)} \times$	$8 \sum_{n=5}^{\infty} + \frac{Ne_n^{(2)}(0)}{Ne_n^{(2)}(0)} \times$
		$\times T_n(\beta, \Omega, \eta_1)^* S_n(\beta, \eta_1)$	$\times T_n(\beta, \Omega, \eta_1) S_n^*(\beta, \Omega, \eta_1)$
	11	$+ \frac{1}{2} F(2\eta_1, -4) e^{i\eta_1} -$	$+ \frac{1}{2} F(2\eta_1, -4) e^{i\eta_1} \}$
26	19	$+ \frac{1}{2} F(2\eta_1, -4) +$	$- \frac{1}{2} F(2\eta_1, -4) +$
	23	$\frac{2}{3} S_2^{(10)}(2\eta_1) \dots$	$\frac{2}{3} S_2^{(10)}(2, \eta_1) \dots$
27	6	$- \frac{1}{60} F(2\eta_1, 8) \dots$	$- \frac{1}{60} F(2\eta_1, -8) \dots$
	14	$- \frac{2}{3} S_2^{(6)}(2\eta_1) \dots$	$- \frac{2}{3} S_2^{(6)}(2, \eta_1) \dots$
30	13	$- \bar{S}_2^{(5)}(\eta_1)$	$- \bar{S}_2^{(5)}(\eta_1) \}$
32	16	$\frac{i\beta^2\Omega}{12} A^*$	$- \frac{i\beta^2\Omega}{12} A^* .$
33	8	$+ 2 e^{6i\eta_1} + 60 \dots$	$+ 2 e^{6i\eta_1} \} + 60 \dots$
	19	$A^* \left[\frac{1}{2} \theta_0 \times$	$+ A^* \left[\frac{1}{2} \theta_0 \times$
	35	$G(\eta_1, 4) (\dots$	$+ G(\eta_1, 4) (\dots$